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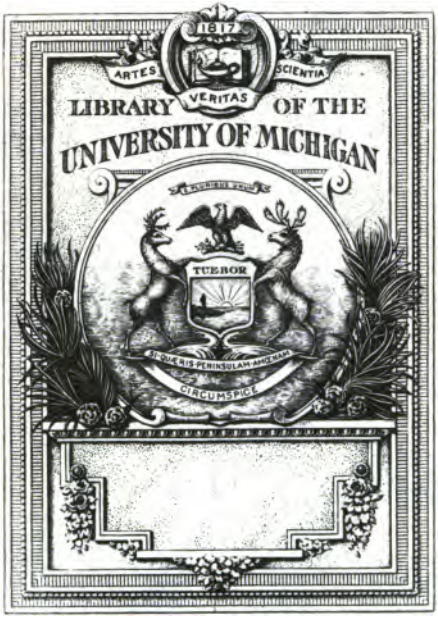
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THE

LEEDS CORRESPONDENT,

A

LITERARY, MATHEMATICAL AND PHILOSOPHICAL

Miscellany;

CONSISTING OF

POETRY, ESSAYS, AND ANECDOTES;

TRANSLATIONS OF LATIN AND FRENCH EXTRACTS;

ANSWERS TO

MATHEMATICAL QUESTIONS, &c. &c.

Communicated by several Correspondents,

AND EDITED BY

JOHN WHITLEY,
WOODHOUSE GROVE, NEAR LEEDS.

> Tarda fluunt ingrataque tempora, quæ spem
> Consiliumque morantur agendi gnaviter id, quod
> Æque pauperibus prodest, locupletibus æque;
> Æque neglectum pueris senibusque nocebit.
>
> HORAT.

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L. III. July 1st 1819.

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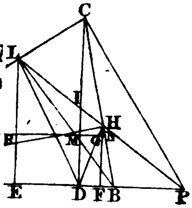

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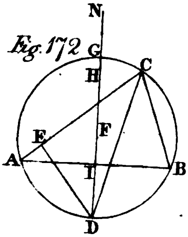

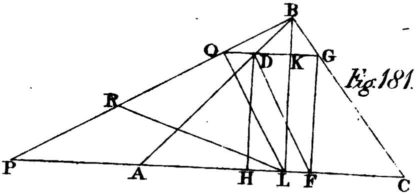

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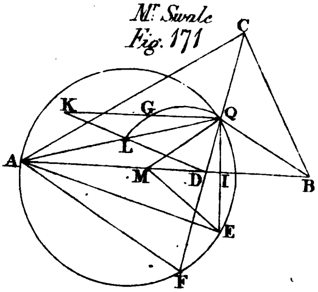

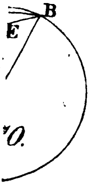

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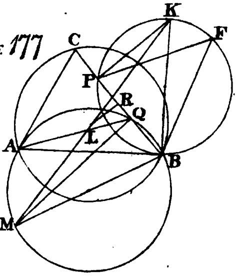

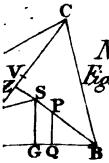

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THE

LEEDS CORRESPONDENT,

A

Literary, Mathematical, and Philosophical Miscellany.

Tarda fluunt ingrataque tempora, quæ spem
Consiliumque morantur agendi gnaviter id, quod
Æque pauperibus prodest, locupletibus æque,
Æque neglectum pueris senibusque nocebit.

HORAT.

No I. JULY, 1819. VOL. III.

Translations are respectfully requested of those, among our junior correspondents, who have not attained to sixteen years of age. The extracts in each language are divided into two or three portions; that any youth, not wishing to translate the whole, may have an opportunity of applying his powers to any single portion.

JUVENILE DEPARTMENT.

Portrait de la Fausse et de la vrai Valeur.

N'ENTENDEZ pas, Messieurs, par le mot de valeur, une hardiesse indiscrète, emportée, qui cherche le danger pour le danger même; qui s'expose sans fruit, et qui n'a pour but que la reputation et les vains applaudissemens des hommes.

Je parle d'une hardiesse sage et réglée qui s'anime à la vue des ennemis, qui, dans le péril même, pourvoit à tout, et prend tous ses avantages; mais qui se mesure avec ses forces; qui entreprend les choses difficiles et ne tent pas les impossibles; qui n'abandonne rien au hazard de ce qui peut être conduit par la vertu: capable enfin de tout oser, quand le conseil est inutile; et prêt à mourir dans la victoire, ou à survivre à son malheur, en accomplissant ses devoirs.

Une Armée.

Qu'est ce qu' une armée? C'est un corps animé d'une infinité de passions différentes, qu'un homme habile fait mouvoir pour la defense de la patrie; c'est une troupe d'hommes armes, qui suivent aveuglement les ordres d'un chef dont ils ne savent pas les intentions: C'est une multitude d'âmes pour la plupart viles et mercenaires, qui, sans songer à leur propre reputation, travaillent à celle des rois et des conquerans; c'est un assemblage confus de libertins qu'il faut assujettir à l'obéissance; de lâches qu'il faut mener au combat; de téméraires qu'il faut retenir; d'impatiens qu'il faut accoutumer à la constance.

* * *

L'Instinct paternel et maternel des Oiseaux.

O TOI, qui follement fais ton Dieu du hasard,
Viens me développer ce nid qu'avec tant d'art,
Au même ordre toujours architecte fidèle,
A l'aide de son bec maçonne l'hirondelle!
Comment, pour élever ce hardi bâtiment,
A-t-elle, en le broyant, arrondi son ciment?
Et pourquoi ces oiseaux si remplis de prudence,
Ont-ils de leurs enfans su prévoir la naissance?
Que de berceaux pour eux aux arbres suspendus!
Sur le plus doux cotons que de lits étendus!
Le père vole au loin, cherchant dans la campagne
Des vivres qu'il rapporte á sa tendre compagne;
Et la tranquille mère, attendant son secours,
Echauffe dans son sein le fruit de leurs amours.
Des ennemis souvent ils repoussent la rage,
Et dans de foibles corps s'anime un grand courage.
Si chèrement aimés, leurs nourrissons un jour
Aux fils qui naîtront d'eux rendront le même amour;
Quand des nouveaux zéphirs l'haleine fortunée
Allumera pour eux le flambeau d'hyménée,

Fidèlement unis par leurs tendres liens,
Ils rempliront les airs de nouveaux citoyens;
Innombrable famille, où bientôt tant de frères
Ne reconnoîtront plus leurs aïeux ni leurs pères.
Ceux qui, de nos hivers redoutant le courroux
Vont se réfugier dan des climats plus doux,
Ne laisseront jamais la saison rigoreuse
Surprendre parmi nous leur troupe parresseuse.
Dans un sage conseil par les chefs assemblé,
Du départ général le grand jour est réglé;
Il arrive, tout part. Le plus jeune peut-être
Demande, en regardant, les lieux qui l'ont vu naître,
Quand viendra ce printemps par qui tant d'exilés
Dans les champs paternels se verront rappelés?

THE EFFECTS OF FRIENDSHIP.

A Translation from the French of Yvon.

FRIENDSHIP has its origin in the insufficiency of our nature; and it is the insufficiency of friendship which is the cause of its destruction. Are we in solitude? We are sensible of our unhappiness; we perceive that we stand in need of help and support; we seek out a person that countenances our inclinations or dislikes, a companion in our pleasures and in our pains; we sigh for one whose whole heart and thoughts will be filled with our welfare. Friendship then seems to be something that possesses the greatest sweetness in the world.—Do we obtain our wishes in this respect? We instantly experience a change in our feelings.

When we perceive at a great distance any thing good, our desires are at first fixed on it; but when we have obtained it, we see nothing in it. It arrests the sight according to its distance; but when the mind can see beyond it, it knows no more repose. Thus that friendship which at a distance is the boundary of all our designs, ceases, on its nearer approach, to act as a limit. It does not fill up the void it had promised to fill. It leaves us in possession of wants, that draw off our attention, and hurry us towards other objects of excellence. We then begin to be negligent and peevish; and soon demand, as a sort of tribute, those acts of courtesy which we had at first received in the form of gifts. It is in the character of the human race, to appropriate to themselves by degrees the very favours which are conferred on them. A long and continued possession naturally habituates men to regard those things as their own which they enjoy from others; and custom persuades them that they have a natural claim on the inclination of their friends: From this they are wishful of producing a title to act in the capacity of their governors. When these pretensions are mutual, as is too often the case, self love is exasperated, excites complaints in both parties, and is the occasion of spleen, indifference, cutting explanations and an open rupture.

We likewise discover occasional defects that were concealed; or we become a prey to those passions which create an aversion in friendship, as violent diseases excite disgust in the sweetest pleasures. Very great men, who are capable of giving the strongest proofs of devotedness, are by no means the best qualified to evince constancy in friendship. We do not at any time find in them that vivacity and solidity which distinguish persons who are timid and grave, and whose well-regulated minds are no strangers to virtue. The delicious and peaceful sensation of friendship is a solace to their hearts, a relaxation and an enlargement to their spirits, and inspires them with greater confidence and gaiety. It partakes of their amusements, entangles itself in their concerns, and mixes with their mysterious pleasures. It is the very soul of their whole existence.

Leeds. JUVENIS.

TIMÆUS, SEU DE UNIVERSO.

De iis igitur, quæ diximus, hæc sit prima distinctio. In omni ratione cum iis rebus, de quibus explicandum videtur esse cognitio. Itaque cum de re stabili et immutabili disputatur oratio talis sit, qualis sit illa, quæ neque redargui, neque convinci potest. Cum autem ingressa est imitata, et efficta simulacra: bene agi putat, si similitudinem veri consequatur. Quantum enim ad id, quod ortum est, æternitas valet: tantum ad fidem veritas. Quocirca si fortè de deorum natura, ortuque mundi disserentes, minùs id, quod habemus, animo consequimur, ut tota dilucidè et planè exornata oratio sibi constet, et ex omni parte secum ipsa consentiat: haud sanè erit mirum: contentíque esse debebitis, si probabilia dicentur. Æquum est enim meminisse, et me, qui disseram hominem esse: et vos, qui judicatis: ut si probabilia dicentur, nil ultrà requiratis.

Quæramus igitur causam, quæ eum impulerit, qui hæc machinatus sit, ut originem rerum, et molitionem novam quæreret. Probitate videlicet præstabat: probus autem invidet nemini: itaque sui similia generavit. Hæc nimirum gignendi mundi causa justissima. Nam cùm constituisset Deus bonis omnibus explere mundum, mali nihil admiscere, quoad natura pateretur: quidquid erat, quod in cernendi sensum caderet, id sibi assumsit, *non tranquillum, et quietum, sed immoderatè agitatum et fluctuans: idque ex inordinato in ordinem adduxit:* hoc enim judicabat esse præstantius. Fas autem nec est, nec unquam fuit, quidquam nisi pulcherrimum facere eum, qui esset optimus. Cum rationem igitur habuisset, reperiebat nihil esse eorum quæ natura cernerentur, non intelligens, intelligente, in toto genere, præstantius: intelligentiam autem ulli rei adjunctam esse, sine animo nefas esse. Quocirca, intelligentiam in animo, animum conclusit in corpore. Sic ratus est opus illud effectum esse pulcherrimum.

Quam ob causam non est cunctandum profiteri, si modò investigari aliquid conjectura potest, hunc mundum animal esse, idque intelligens, et divina providentia constitutum. Hoc posito, quod sequitur, videndum est, cujúsnam animantium Deus in fingendo mundo similitudinem secutus sit. Nullius profectò, eorum quidem, quæ sunt nobis nota animantia: sunt enim omnia in quædam genera partita, aut inchoata, nulla ex parte perfecta; imperfecto autem, nec absoluto simile, pulcrum esse nihil potest: cujus ergo omne animal quasi particula quædam est, sive in singulis, sive in diverso genere cernatur; ejus similem mundum esse dicamus.

Omnes igitur, qui animo cernuntur, et ratione intelliguntur animantes, complexu rationis, et intelligentiæ, sicut homines, hoc mundo, et pecudes, et omnia, quæ sub adspectum cadunt, comprehenduntur: quod enim pulcherrimum in rerum natura intelligi potest, et quod ex omni parte absolutissimum est, cum Deus similem mundum efficere vellet, animal unum, adspectabile, in quo omnia animalia contineremus, effecit. Rectène igitur unum mundum dixerimus, an sint plures, an innumerabiles, dictu verius, et melius? Unus profectò, si quidem factus est ad exemplum. Quod enim omneis animanteis eos, qui ratione intelliguntur, complectitur, id non potest esse cum altero.

Rursus enim alius animans, qui eum contineat, sit necesse est, cujus partes sint, animantes superiores, cælúmque hoc, simulacrum illius ultimi sit, non proximi. Quorum ne quid accideret, atque ut hic mundus esset animanti absolutum simillimus hoc ipso, quod solus atque unus esset, idcirco singularem Deus hunc mundum, atque unigenum procreavit. Corporeum autem, et adspectabile, itémque tractabile omne necesse est esse, quod natum est. Nihil porro, igni vacuum, videri potest, nec vero tangi, quod careat: solidum autem nihil, quod terræ sit expers. Quamobrem mundum efficere moliens Deus, terram primùm, ignemque jungebat.

Omnia autem duo ad cohærendum, tertium aliquid requirunt, et quasi nodum vinculumque desiderant. Sed vinculorum id est aptissimum, atque pulcherrimum, quod ex se, atque de his quæ adstringit, quam maximè unum efficit. Id optimè assequitur, quæ Græcè *analogia*, Latinè, (audendum est enim, quoniam hæc primùm à nobis novantur,) comparatio, proportióve dici potest. Quando enim trium, vel numerorum, vel figurarum, vel quorumcunque generum contingit, ut, quod medium sit, uti primum proportione, ita id postremò comparetur, vicissimque ut extremum cum medio, sic medium cum primo conferatur: id quod medium est, tum primùm sit, tum postremum: postrema verò et prima media fiunt. Ita necessitas cogit, ut eadem sint ea, quæ devincta fuerunt. Eadem autem cùm facta sint, efficitur, ut omnia sint unum. Quod si universitatis corpus, planum, et æquabile explicaretur, neque in eo quidquam esset requisitum: unum interjectum medium, et seipsum, et ea, quibus esset interpositum, colligaret. Sed cùm soliditas mundo quæreretur: solida autem omnia uno medio nunquam, duobus semper copulentur, ita contingit, ut inter ignem, et terram, aquam Deus, animámque poneret, eáque inter se compararet, et proportione conjungeret, ut quemadmodum ignis, animæ: sic anima, aquæ: quódque anima aquæ; id aqua, terræ proportione redderet.

De Ostendæ obsessione.

Area parva Ducum, totus quam respicit orbis,
Celsior una malis, et quam damnare ruinæ
Nunc quoque fata timent, alieno in littore resto.
Tertius annus abit; toties mutavimus hostem.
Sævit hiems pelago, morbisque furentibus æstas;
Et nimium est quod fecit Iber crudelior armis.
In nos orta lues: nullum est sine funere funus;

[illegible]

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[illegible]

same; the other being destitute of reason comes to our minds through the senses; and all that is perceivable in this way is derived and perishes, nor can it have a permanent existence. But all that is derived, must be produced from some cause; for the origin of nothing can be ascertained when the cause is removed. Wherefore if he who meditates some production shall see that form of things which is always the same, and shall propose it to himself as a model, he must produce something excellent in its nature; but if he should propose that which is derived, he will never reach that beauty at which he aims. All this then we may denominate "heaven" or "the world," or by any other eligible term. Respecting which we should first consider, what ought to be considered first in all such discussions, whether it has always existed as having been derived from no origin, or has been produced, or has commenced at some period, since it is visible and tangible, and every where invested with body. But all such things strike the senses, and, coming within the range of the senses, belong to that class of things which we have said have an origin and are derived, as nothing can be produced without a cause. But it is indeed difficult to discover him who is as it were the Parent of this universe, and, when you have made the discovery, it is wrong to make it public. Now we must see what model was followed by the Architect of this vast fabric, whether that which is always one and the same and like himself, or that which we maintain was derived and had a beginning. But if this world is beautiful, if its Maker is good, he would undoubtedly prefer the imitation of that form of things which bears the impress of eternity, or if otherwise, (which it would be impious to assert!) he would have followed an imperfect pattern, instead of that which is eternal. It is not therefore doubtful, that he would prefer what is stamped with eternity, since there is nothing more beautiful than the world or more excellent than its Creator. Thus then being produced, it was made in conformity with the dictates of intelligence and wisdom, and has no limit but unchangeable eternity. From which we come to the conclusion,—that this world which we behold must be the eternal resemblance of some eternal form. But, in every rational disquisition the exordium is the most difficult.

Idle. **WILLIAM VINT.**

* * *

Dulces ante omnia Musæ. VIRG.

THE VILLAGE SCHOOL.

BEHOLD the village-boy on yonder green,
Diffusive health seems bursting o'er his mien!
With hard brown crust and task-book in his hand,
He runs about to call his sportive band:
The band appears, away to school they hie,
The grammar, task, or puzzling rule to ply.

Swift now they bound, impetuous, o'er the plain,
Health, joy, and gladness, mingling in the train.
One throws a stone against the nearest door,
Away they scour, confus'd, across the moor;
That gains the dell, this steals behind the wall,
In secret ambush lurk the yonkers all.
Out comes the owner, staring far and wide,
The infant, too, comes waddling by his side.
The smirking boys in ambush lie unseen,
While he with rapid eye explores the green:
Then turning round, he, with a wistful gaze,
The neighb'ring field and ev'ry nook surveys.
No human form appears upon the plain,
Save the poor rustic, or the busy swain,
Who, doom'd for healthy exercise to toil,
Wields the sharp axe, or tills the swarthy soil.

Soon as he shuts the clapping door again,
On ev'ry side, upstarts the laughing train.

Forth rush the boys, enraptur'd o'er the green,
And, fast and far, their rivals follow keen;
Each boasting loud, how fearless he'd have stood,
If the wrong'd neighbour had the band pursued.
Disporting thus, they linger in the way,
Nor think of school—they're all resign'd to play.

But hark! the bell now sounds the dread alarm,
In thick disorder round the door they swarm:
That takes the end, and this the middle space,
Off come the hats, and each resumes his place.
What sounds discordant issue from the door!
A hundred tongues their tasks are conning o'er.

Now one appears more bold than all the gang,
Leaps on the seat, with hard and heavy clang;
By him unseen, now flies the hated cane,
His flushing blood quick starts in ev'ry vein.
He stalks along with downcast eyes and slow,
And, with a frown, receives th' expected blow.

Another, with a sly and harmless look,
Demurely peeps around, and tears his book.

Solemn and silent, as if crost full sore,
This stands in piteous plight behind the door;
Puzzled by grammar, and his head cast down,
He mutters o'er his adverb or his noun,
Then full of mirth around him casts his eye,
Or calls another that stands ling'ring by.

Another, still more busy than the rest,
Looks archly round, with mischief in his breast;
And with his knife he nicks the desk again,
Until the erring steel his fingers stain.
With wrinkled brow, he views the deep gash bleed,
And, scowling, cries "Some other did the deed."

In vain, with longing eyes they turn to see
The welcome signal that's to set them free.
At length 'tis giv'n,—from ev'ry seat they rise,
And fill with rapt'rous shouts the spreading skies.
Fast from the door the mingling tumult steals,
And ev'ry breast a new emotion feels.
Swift bounds the village-boy across the plain,
And seeks with joy his wonted home again.

A. FAWCETT, aged 13 years.

Wortley near Leeds, April 24, 1819.

THE BAGPIPES.

The following humourous description of bagpipe-music and its effects, is extracted from the second edition of that entertaining mock-heroic poem, ANSTER FAIR, the composition of Mr. James Tennant, a schoolmaster, near Edinburgh. The reader will perceive in these verses an ingenious attempt to give the origin of two famous Scottish tunes.

His hollow drone, with mouth wide-gaping, lay
  Over his shoulder pointing to the sky,
Ready to spue its breath, and puff away
  The lazy silver clouds that sit on high:
His bag swell'd madly to begin the play,
  And with its bowel-wind groan'd inwardly;
Not higher heav'd the wind-bags, which of yore
Ulysses got from him who rul'd th' Æolian shore.

He thus the King with reverence bespoke:
  "My liege, since Heav'n with bagpipe-levell'd fire
Hath turn'd my Brethren's gear to dust and smoke,
  And testify'd too glaringly its ire,
It fits me now, as yet my bagpipe's poke
  Remains unsing'd, and every pipe entire,
To play my tune—O King, with your good will—
And to the royal ear to prove my piping skill."

Nodded his liege assent, and straightway bade
  Him stand a-top o' th' hillock at his side;
A-top he stood; and first a bow he made
  To all the crowd that shouted far and wide;
Then, like a Piper dext'rous at his trade,
  His pipes to play adjusted and apply'd;
Each finger rested on its proper bore,
His arm appear'd half-rais'd to wake the bag's uproar.

A space he silent stood, and cast his eye
  In meditation upwards to the pole,
As if he pray'd some fairy pow'r in sky
  To guide his fingers right o'er bore and hole;
Then pressing down his arm, he gracefully
  Awak'd the merry bagpipe's slumb'ring soul,
And pip'd and blew, and play'd so sweet a tune,
As might have well unspher'd the reeling midnight
    moon.

His ev'ry finger, to its place assign'd,
  Mov'd quiv'ring like the leaf of aspen tree,

Now shutting up the skittish squeaking wind,
Now op'ning to the music passage free;
His cheeks, with windy puffs therein confin'd,
Were swoln into a red rotundity,
As from his lungs into the bag was blown
Supply of needful air to feed the growling drone.

And such a potent tune did never greet
The drum of human ear with lively strain;
So merry, that from dancing on his feet
No man undeaf could stockishly refrain;
So loud, 'twas heard a dozen miles complete,
Making old Echo pipe and hum again,
So sweet, that all the birds in air that fly,
Charm'd into new delight, come sailing through the sky.

Crow, sparrow, linnet, hawk, and white-wing'd dove,
Wheel in aërial jigg o'er ANSTER loan;
The sea-mews from each Maian cleft and cove
O'er the deep sea come pinion-wafted on;
The light-detesting bats now flap above,
Scaring the sun with wings to day unknown,
Round ROBERT's head they dance, they cry, they sing,
And shear the subtil sky with broad and playful wing.

Then did th' infectious hopping-mania seize
The circles of the crowd that stood more near,
Till, round and round, far spreading by degrees,
It madden'd all the loan to kick and rear;
Men, women, children, lilt and ramp, and squeeze,
Such fascination takes the gen'ral ear!
Ev'n babes, that at their mothers' bosoms hung,
Their little willing limbs fantastically flung.

And cripples from beneath their shoulders fling
Their despicable crutches far away,
Then, yok'd with those of stouter limbs, up-spring
In hobbling merriment, uncouthly gay;
And some on one leg stand y-gamboling;
For why? The other short and frail had they;
Some, whose both legs distorted were and weak,
Dance on their poor knee-pans in mad prepost'rous freak.

So on they tript, King, MAGGIE, Knight and Earl,
Green-coated courtier, satin-snooded dame,

Old men and maidens, man, wife, boy, and girl,
The stiff, the supple, bandy-legg'd, and lame,
All suck'd and rapt into the dance's whirl,
Inevitably witch'd within the same:
Whilst ROB, far-seen, o'erlooks the huddling loan,
Rejoicing in his pipes, and squeals serenely on.

But such a whirling and a din there was,
Of bodies and of feet that heel'd the ground,
As when the Maelstrom in his craggy jaws
Engluts the Norway waves with hideous sound;
In vain the black sea-monster plies his paws
'Gainst the strong eddy that impels him round;
Rack'd and convuls'd, the ingorging surges roar,
And fret their frothy wrath, and reel from shore to
shore.

Then was the shepherd, that on Largo-law
Sat idly whistling to his feeding flock,
Dismay'd, when looking south-eastward he saw
The dusty cloud more black than furnace-smoke;
He lean'd his ear, and catch'd with trembling awe
The dance's sounds that th' ambient ether broke;
He bless'd himself, and cry'd, "By sweet St. John!
The devil hath got a job in ANSTER's dirty loan."

At length the mighty Piper, honest ROB,
His wonder-working melody gave o'er,
When on a sudden all the flouncing mob
Their high commotion ceas'd, and toss'd no more;
Trunk, arm, and leg, forgot to shake and bob,
That bobb'd and shak'd so parlously before;
On ground, fatigu'd, the panting dancers fall,
Wond'ring what witch's craft had thus embroil'd
them all.

But from that hour, the Monarch and the mob
Gave MAGGIE LAUDER's name to ROBERT's tune,
And so shall it be call'd, while o'er the globe
Travels the waning and the crescent moon.
And from that hour the puissant Piper ROB,
Whose bagpipe wak'd so hot a rigadoon,
From his well-manag'd bag, and drone, and chanter,
Obtain'd the glorious name of Mighty ROB the RANTER.

CHARACTER OF CLEANDER.

Nec aspera torrent.

This essay is a juvenile production, by the REV. ROBERT HALL, A. M. of Leicester. His excellences as an elegant and nervous writer, are universally acknowledged; and the only regret which any one feels on reading his works, is, that a man thus eminently qualified to instruct and delight the age, should be so regardless of his own fame, and the gratification of his readers, as to be sparing of his publications. We have petty triflers and imitators in abundance; but how few are the original writers like ROBERT HALL!

It is extracted from a pamphlet printed for private distribution, which contains all his juvenile pieces that can be traced and identified.

WHOEVER contemplates the various calamities that fill the world, and the still more numerous avenues by which we are exposed to distress, will be affected with a sense of the misery of man: in this survey, we need not search for remote and distant evils; we need not crowd our imagination with the horrors of war, the progress of armies, or the desolation of states; in the most familiar walks of life we meet with scenes at which humanity must bleed. Scenes of distress lie open on every side; every quarter is filled with the groans of the dying, and lamentations for the dead. In the mass of mankind we can scarcely select an individual, in whose bosom there does not rankle unpublished griefs; and could we look into the hearts of the most tranquil, we should often find them a prey to unpitied regrets, torn with anxiety, and bleeding with disappointments.

Retiring from this melancholy spectacle without looking any further, we might be ready to consider the world as a great nursery of disease, a vast receptacle of miseries, filled with beings whom Providence has endowed with sensibility to suffer, rather than capacities to enjoy; but to him who views the moral influence of afflictions, the evils they are intended to correct, and the benefits they impart, they will appear in a very different light: he will consider them at once the punishments of vice, and the cure of it. Sorrow is, indeed, the offspring of guilt, but the parent of wisdom. Stern in her aspect, and severe in her deportment, she is, however, sent on a message of mercy. She is destined to follow in the footsteps of intemperance, to break her enchantments, to expose her delusions, and to deliver from thraldom such as are entangled in her snares, or are sleeping in her arms. Whoever surveys the course of his past life, with a view to remark the false steps he has taken in it, will find, that as they have been preceded by indiscretion, they have been recalled by distress. To every object our attachment is proportioned to the pleasures we have received, or expect to receive from it; and the passion will continue to be cherished, as long as the recollection of it calls up ideas of pleasure rather than of pain. Now, every vicious pursuit is founded on indulgence, and disguised by inclination. To the licentious and the abandoned, therefore, there is no prospect of the termination of their vices, till, by actual experience of the miseries they inflict, they convey to the mind more sentiments of aversion than of love.

From that moment the enchantment is dispelled, the false colours stripped off, they will be regarded as specious deformities, and real dangers. Multitudes, who could never be persuaded by the calls of interest or the voice of conviction, to restrain the license of their passions, and abandon their criminal pursuits, have been reclaimed by the lash of adversity. The decay of health, the desertion of friends, and the neglect of the world, have not unfrequently softened those hardier spirits, to whom the charms of virtue have been displayed in vain.

Nor is sorrow less effectual in the correction of foibles, than in the extinction of vice. Cleander, in other respects a man of virtue and honour, had from his infancy accustomed himself to the unbounded indulgence of his tongue. Upon all occasions he trod on the very brink of decorum, a total stranger to the delicacies of friendship, which generously hides the faults it cannot correct. His ridicule was turned on the imperfections of his friends and his enemies, with indiscriminate severity. The splendour of distinguished virtue, which sets at a distance the reproaches of the world, and almost sanctifies the blemishes of an illustrious character, exempted no foibles from the scourge of Cleander; but rather quickened his acuteness to remark, and his asperity to expose them, as it furnished a display of his penetration in discovering imperfections, where there appeared to the world nothing but unmingled excellence. It was, indeed, his delight to remark the shades of a brilliant character, and to pourtray with exactness the secret gradations of excellence, by which it fell short of perfection. Yet, in Cleander, this conduct by no means sprang from envy of superior worth, or the malignant desire of degrading every one to his own level. He possessed the magnanimity of a virtuous mind, and disdained to lessen his own inferiority by any other means than that of honest emulation. It had its basis in a taste for ridicule, and the pride of wit. His deportment could not fail to issue in perplexity and distress. His enemies considered him as a kind of beast of prey, a savage of the desert, whom they were authorised to wound by every weapon of offence, some by open defamation, and some by poisoned arrows in the dark. His friends began to look upon him with alienation and distrust, esteeming their character too sacred to be suspended for the sport of an individual, on the breezy point of levity and wit.

His appearance was a signal for general complaint; and he could scarcely enter into company hoping to enjoy the unmingled pleasures of social converse, but he had innumerable jealousies to allay, and misunderstandings to set right. He was every where received with marks of disgust; met with resentment for which he could not account; and was daily obliquely insulted for careless strokes of satire, of which he retained no recollection. Wherever he turned himself, he found his path was strewed with thorns; and that even they who admired his wit, secretly vilified his character, and shrunk from his acquaintance. His peace began to bleed on every side; his reputation was tarnished; his fairest prospects blasted; and Cleander, at length, awakened from his delusions, was convinced, when it was too late, of a lesson he had often been taught in vain; that the attachments of friendship, and the tranquility of life, are too valuable to be sacrificed to a blaze of momentary admiration.

A consideration of the benefit of afflictions, should teach us to bear them patiently, when they fall to our lot; and to be thankful to heaven for having planted such barriers around us, to restrain the exuberance of our follies and our crimes.

Let these sacred fences be removed; exempt the ambitious from disappointment, and the guilty from remorse; let luxury go unattended with disease, and indiscretion lead into no embarrassments or distresses; our vices would range without controul, and the impetuosity of our passions have no bounds; every family would be filled with strife, every nation with carnage, and a deluge of calamities would break in upon us, which would produce more misery in a year, than is inflicted by the hand of providence in a lapse of ages.

A DISSERTATION ON FORKS.

At present forks are so necessary at table among polished nations, that the very idea of eating a meal without them excites disgust. The introduction of them, however, is of so modern a date that they have scarcely been in use three centuries. "At such a short distance are we from the origin of many things," says Pliny, in speaking of a thing which, though very new, was then exceedingly common. Neither the Greeks nor the Romans have any name for these instruments; and no phrase or expression which, with the least probability, can be referred to the use of them, occurs any

* * *

where in their writings. But had forks been known, this could not have been the case, since so many entertainments are celebrated by the poets or described by other writers; and they must also have been mentioned by Pollux, in the very full catalogue which he has given of articles necessary for the table.

The Greek word *creagra* signified indeed a fork, but not a fork used at table. It meant merely a flesh-fork, or that instrument employed by cooks to take meat from a boiling pot, as is proved by the connexion of the words in all those passages where it occurs. It is mentioned by Pollux, and by Anaxippus, in Athenæus, among the utensils of the kitchen; and the scholiast on Aristophanes says, that this fork had a resemblance to the hand, and was used to prevent the fingers from being scalded. Suidas quotes a passage where the word denotes a hook at the end of a long pole, with which people, even at present, draw up water-buckets from wells and other deep places. This instrument, therefore, appears sometimes to have had only a hook, but sometimes two or more prongs. *Creagra* occurs once in Martianus Capella, a Latin writer, but in a passage which is not intelligible.

Equally inapplicable to our forks are the words *furca, fuscina, furcilla, fuscinula,* and *gabalus,* which are given in dictionaries. The first two were undoubtedly instruments which approached nearly to our furnace and hay forks. The trident of Neptune also was called *fuscina.* The *furcilla* even was large enough to be employed for a weapon of defence, as is proved by the expressions *furcillis ejicere* and *expellere. Fuscinula,* which in modern times is used chiefly for a table fork, is not to be found even once in any of the old Latin writers. The old translation of the Bible only explains the word *creagra* by *fuscinula. Gabalus,* according to every appearance, has given rise to the German word *gabeln,* but it denotes the cross or gallows, which last word Vossius deduces from it.

A learned Italian, who asserts also that the use of forks is very new, is of opinion, that the Romans often used *ligulæ* instead of forks. This I shall not deny; but the *ligula* certainly had more resemblance to a small spatula or tea-spoon, than to our forks. According to Martial, many spoons at the other end seem to have been *ligulæ*. Thus

(Lib. xiv,) Epigr. 120. *Ligula Argentea.*

Quamvis me ligulam dicant equitesque patresque,
Dicor ab indoctis lingula grammaticis.

121. *Cochlearia.*

Sum cochleis habilis, sed nec minus utilis ovis;
Num quid scis potius cur cochleare vocer?[*]

But the two epigrams must be read in conjunction, so that the second may appear a continuation of the first; for the epithets *habilis* and *utilis* can be applied to no other term than *ligula*. Besides, it is certain that the titles of the epigrams, or at least the greater part of them, were not added by the poet but by transcribers. The name also, which originally was *lingula*, gives an idea of the form. We read likewise that this instrument was used for scumming, for which purpose nothing is less fit than a fork.*

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[*] Uniting the two epigrams, according to the directions of our author, we have the following connected couplets. The "unlearned grammarians" to whom Martial alludes, were in those days the sticklers for analogy and derivation; and because the slightly-scooped spatula or slicer had the appearance of a little tongue, they insisted that it ought not to be called *ligula* "a small spoon or spatula," but *lingula*, from *lingua* "a tongue," which in shape it resembled.

ON A SILVER SLICER, OR A SMALL SPOON.

A SPOON or SLICER is the name
Which from great lords and knights I claim;
Yet weak grammarians do me wrong
By styling me "the little tongue."

Cockle-shells with ease I drain,
And eggs-meat does not long remain;
Why should I not be call'd as soon
"An egg-drain" as "a cockle-spoon?"

ED.

I have, I know not how, a great unwillingness to represent the table of our ancestors as without forks; yet this was certainly the case: and when we reflect on their manner of eating, it will readily be perceived that they could much easier dispense with the use of them than we can. All their food, as is still customary in the East, was dressed in such a manner as to be exceedingly tender, and therefore could be easily pulled to pieces. It appears, however, that people, though not in the earliest periods, employed the same means as our cooks, and suffered meat to lie some time that it might be easier dressed. We often read that cooks, in order to provide an entertainment speedily, will kill an animal, and having cleaned and divided it, roast it immediately, and then serve it up to their guests. But it is well known that the flesh of animals newly killed, if cooked before it has entirely lost its natural warmth, is exceedingly tender and savoury, as we are assured in many books of travels.

Formerly all articles of food were cut into small morsels before they were served up; and this was the more necessary, as the company did not sit at table, but lay on couches turned towards it, consequently could not well use both their hands for eating. For cutting meat, persons of rank kept in their houses a carver, who had learned to perform his duty according to certain rules, and who was called *scissor, carpus, carptor*, and by Apuleius is named *diribitor*. This person used a knife, the only one placed on the table, and which in the houses of the opulent had an ivory handle, and was commonly ornamented with silver.*

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* "That the *ligula* was smaller than the *cochlear* is proved by Martial, viii. 23." This remark by Professor Beckmann is erroneous, as will be evident to any one who will look at the four closing verses of that epigram, in which the terms are in a regularly decreasing series. If the *ligula* was "a spoon," the *cochleare* was less, and must have been a desert-spoon. The English reader may gather the meaning of them from the following translation:

Thy present, Paulus!—was it not
A handsome silver-plated *pot*?
Why shouldst thou so much cash expend?
A *slicer* was as fit to send.
A *desert-spoon* had been enough,
Or any still more common stuff;
Or had it been a *cockle-shell*,
Or *nought* at all,—'twas quite as well.

ED.

Bread also was never cut at table. In former times it was not baked so thick as at present, but rather like cakes, and could easily be broken; hence mention is so often made of the breaking of bread. Juvenal, when he wishes to describe old bread, does not say that it could not be cut, but that it could not be broken. The ancient form of bread is still retained in the paschal cake of the Jews, and in the *knæckbröd* of the Swedes. The latter, which is almost as brittle but not so tender as biscuit, is not cut when used, but broken.

The Chinese, who also use no forks, have however some small sticks of ivory, which are often of very fine workmanship, and inlaid with silver and gold. A couple of these is placed before each guest, who employs them for putting into his mouth the meat which has been cut into small bits. But even this resource was not known two centuries ago in Europe, where people, as is still done by the Turks, every where used their fingers. As a proof, I shall not quote passages where mention is made of persons putting their hands or fingers into the dish; for such a mode of speaking is yet employed, though forks, as is well known, are in common use. I shall refer only to one passage in Ovid, which admits of no doubt, and where the author would certainly have mentioned these instruments, or rather have communicated to his pupils in the art of love a precept which at present is given to children, had the former been taught when young how to make use of forks.

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*Posidonius relates in Athenæus, iv. 13. p. 154, that the Gauls used to take roast meat in their hand and tear it to pieces with their teeth, or cut it with a small knife which each carried in his girdle. This was told as a thing uncommon to the Greeks. Baumgarten adds, that Posidonius said also that the Gauls had bread so flat and hard that it could be easily broken. But this circumstance I cannot find in Athenæus.

> Carpe cibos digitis; est quiddam gestus edendi;
> Ora nec immunda tota perunge manu.*

Had they been used by the Romans, they must necessarily have occurred among the numerous remains of antiquity which have been collected in modern times. But Baruffaldi and Biörnstähl, who both made researches respecting them, assure us, that they were never able to find any. Count Caylus and Grignon only, assert the contrary. The former has given a figure and description of a silver two-pronged fork, which was found among rubbish in the Appian way. It is of exceedingly beautiful workmanship, and at one end terminates in a stag's foot. Notwithstanding the high reputation of this French author, I cannot possibly admit that every thing of which he has given figures is so old as he seems to imagine. Grignon found in the ruins of a Roman town in Champagne some articles which he considers as table forks; but he merely mentions them, without giving a description sufficient to convince one of the truth of what he asserts, which, in regard to a thing so unexpected, was certainly requisite. One fork was of copper or brass; two others were of iron; and he says, speaking of the latter, that they seem to have served as table forks, but were coarsely made. I however doubt whether he conjectured right in regard to the use of them.

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* Seize with thy fingers all thy food,
And shew thy pow'rs at eating good;
Thy dirty hands across thy face
Move not, for fear of sad disgrace.

Ed.

As far as I know, the use of forks was first known in Italy towards the end of the fifteenth century; but at that time they were not very common. Galeotus Martius, an Italian, resident at the court of Matthias Corvinus, king of Hungary, who reigned from 1458 to 1490, relates in a book which he wrote in regard to the life and actions of this prince, that in Hungary, at that time, forks were not used at table, as they were in many parts of Italy,* but that at meals each person laid hold of the meat with his fingers, and on that account they were much stained with saffron, which was then put into sauces and soup. He praises the king for eating without a fork, yet conversing at the same time and never dirtying his clothes.

That in France, at the end of the sixteenth century, forks even at court were entirely new, is proved by a book already quoted in a preceding volume of this work, entitled *l'Isle des Hermaphrodites.*[*] It will, therefore, excite no wonder that in the same century forks were not used in Sweden.

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* This work has been twice printed.—"They have a custom of all eating out of one large dish; not like us, each of whom takes a separate plate. Neither does any one in that country, when he receives a mouthful or bites at the flesh, make use of a fork, which is now the general fashion in that part of Italy which is situated beyond the Po. For every one who has a large biscuit placed before him, takes out of the common dish whatever he chooses; and placing it on his wheaten trencher, he cuts it into little pieces, and puts them singly into his mouth. Carvers of meat are men of uncommon occurrence among the Hungarians; on this account it is with the greatest difficulty that any person can preserve his hands or his clothes from becoming stained and dirty, in that abundance which is usual at their entertainments. For the saffron sauce often oozes out by drops, and makes a man filthy. They employ saffron on all occasions in great plenty. No one therefore will wonder at their being stained by the sauce and by saffron preparations, their fingers and nails in particular by which they lay hold of their food. King Matthias handled every thing, but he never stained himself at all, though he was occupied the whole time with discourse. I beheld in astonishment, with my own eyes, the king either speaking, or paying the greatest attention to some poem or conversation, and yet continuing to handle his food and keep himself free from the least stain. This was indeed wonderful, when other people, who employed the utmost carefulness and diligence, were not able to preserve their hands and their garments clean and unspotted."

But it must appear very strange that Thomas Coryate, the traveller, should see forks for the first time in Italy, and in the same year be the first person who used them in England, on which account he was called, by way of joke, *Furcifer.*

Coryate in the year 1608 travelled, for five months, through France, Italy, Swisserland, and a part of Germany. An account of this tour was published by him, in 1611, under the singular title of *Crudities,* a new edition of which appeared in 1776. The passage to which I allude I shall quote from the original; but I must first remark that this Englishman travelled afterwards to the East Indies, and in 1615 wrote in that country some letters which may be seen in *Purchas his Pilgrims,* vol. ii; also in the edition of the *Crudities* published in 1776. A more particular account of this singular character may be found in the *Biographia Britannica* second edit. vol. iv, p. 273. In page 90 of the Crudities, the author says: "Here I will mention a thing that might have been spoken of before in discourse of the first Italian town. I observed a custom in all those Italian cities and towns through which I passed, that is not used in any other country that I saw in my travels, neither do I think that any other nation of Christendom doth use it, but only Italy. The Italians, and also most strangers that are commorant in Italy, do always at their meals use a little fork when they cut their meat. For while with their knife (which they hold in one hand) they cut the meat out of the dish, they fasten their fork, which they hold in their other hand, upon the same dish; so that whatsoever he be that sitting in the company of any others at meals, should unadvisedly touch the dish of meat with his fingers from which all at the table do cut, he will give occasion of offence unto the company, as having transgressed the laws of good manners, insomuch that for his error he shall be at least browbeaten, if not reprehended in words. This form of feeding, I understand, is generally used in all places of Italy; their forks being for the most part made of iron or steel, and some of silver, but those are used only by Gentlemen. The reason of this their curiosity is, because the Italian cannot by any means indure to have his dish touched with fingers, seeing all men's fingers are not alike clean. Hereupon I myself thought good to imitate the Italian fashion by this forked cutting of meat, not only while I was in Italy, but also in Germany, and oftentimes in England since I came home, being once quipped for that frequent using of my fork by a certain learned Gentleman, a familiar friend of mine, one Mr. Laurence Whitaker, who in his merry humour doubted not to call me at table *Furcifer*, only for using a fork at feeding, but for no other cause."

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[*] "They never touch their food with their hands; but convey it by means of forks up to their mouths, at the same time stretching out their necks and their bodies over their plates, which are very often changed for them. Even their bread is divided into slices, without their being at the trouble of cutting it; and I believe that they would be very glad if any one would hereafter invent a method of chewing their victuals without giving them the trouble of performing that operation. From what I have been able to perceive of them, the circumstance of their having among them abundance of artificial teeth would be a source of torment to them, because they would have to be removed before they could sit down at table." Cologne, 1724

In many parts of Spain, at present, drinking-glasses, spoons, and forks are rarities; and even yet, in taverns, in many countries, particularly in some towns of France, knives are not placed on the table, because it is expected that each person should have one of his own; a custom which the French seem to have retained from the old Gauls. But as no person would any longer eat without forks, landlords were obliged to furnish these, together with plates and spoons.

Among the Scots highlanders, as Dr. Johnson asserts, knives have been introduced at table only since the time of the Revolution. Before that period every man had a knife of his own as a companion to his dirk or dagger. The men cut the meat into small morsels for the women, who then put them into their mouths with their fingers. The use of forks, at table, was at first considered as a superfluous luxury; and, therefore, they were forbidden to convents, as was the case in regard to the congregation of St. Maur.

The English, Dutch, and French have adopted the Italian names *forca* and *forchella*, given to our table forks; though these appellations, in my opinion, were used at an earlier period, to denote large instruments, such as pitch-forks, flesh-forks, furnace-forks; because in the low German, *forke* is a very old name given to such implements. The German word *gabel*, which occurs first in dictionaries for these large instruments, is of great antiquity, and has been still retained in the Swedish and Dutch. It appears to have been used for many things which were split or divided into two; at any rate, it is certain that it is not derived from the Latin word *gabulus*. *Beckmann's Inventions*.

To the Editor of the Leeds Correspondent.

FEMALE MATHEMATICIANS.

To your work, Sir, and to others like it, the honour belongs of introducing into society a relish for mathematical and scientific amusements. Not only do the names enumerated in your pages bear testimony to the individuals benefitted by your labours; but there are others, whose personal diffidence, or recent attempts in this line of study, prevent them from appearing as occasional contributors. Nor does this honour apply exclusively to the lords of the creation; Many of the fair sex have become enamoured of these studies; and it is my happiness to communicate to you the information, that a considerable change has taken place in one of the most genteel female circles in a neighbouring town, by adopting philosophical and mathematical amusements in their select parties.

Who is ignorant of the influence of fashion among ladies? Well; unlike many other matters of the greatest utility, this improvement had its origin in the vogue or fashion of the day, and was introduced by a London lady lately married into this neighbourhood. While a resident in town, it was her practice to frequent one of the scientific Institutions in the Metropolis, and to attend several of the lectures which were delivered on different subjects. In this way she acquired a taste for Chemistry, and soon afterwards became attached to the kindred study of Geology. To her previous attainments she has now added a knowledge of Geometry; and it was on one occasion, when she was displaying in confidence to an intimate friend some ofthe delicious recreations that were couched under the unimposing symbols of lines and angles, her friend was filled with admiration and requested of her as a particular favour, to be initiated in these charming mysteries. The desire was no sooner expressed than the good lady attempted to gratify it, by lending her friend an elementary treatise on Geometry. This was soon conquered; and the two ladies have since that time travelled together over the first six books of "Euclid's Elements," and over the two quarto volumes of "Sanderson's Algebra." By this specimen you will judge that theirs are not superficial attainments. They have been the authors of a better order of things in this neighbourhood. The lady who was the pupil of the other, had been passionately fond of cards; but since her acquaintance with the wonders of Geometry and Algebra, all her fervour for cards has abated; and her ardour for the Mathematics, proceeding in a contrary direction, has increased with the pleasure which she derives from her new studies. Being yet young and unmarried, she has had leisure and opportunity for making many proselytes to the same cause; and has inspired into several of her own age and sex much of this noble enthusiasm.

The mathematic-mania, all lovely as it is, has infected many an amiable woman. Its tendency, we all know, is to enlarge, ennoble and refine the human intellect; and is it not now generally acknowledged, that women are endowed by Heaven with as large a portion of intelligence as men? The mode in which they were formerly educated, their situation in life and domestic circumstances, have militated against their improvement, and afforded them but few opportunities of acquiring that knowledge or experience in the world which is one great means of fostering talent and of eliciting it. In my province of schoolmaster, I can bear testimony to the powers of the female capacity, which are, in the subjects that have come under my observation, in every respect equal to those of the hardier sex. My prepossessions would incline me to say, that their aptitude for receiving instruction is much greater. But such a remark might affright a timid youth, like your former entertaining correspondent *Timothy Triangle*, and deter him from seeking out a wife among "learned ladies." The most common female education is not now considered to be in any tolerable degree complete, without a knowledge of Grammar and Logic. These solid foundations for higher acquisitions, will have a salutary effect on the young men of this age. They will find, when they pay their devoirs to the fair sex, that they are not addressing the flippant and illiterate beings which their imaginations had painted. To secure their affections, they must themselves evince a reach of intellect corresponding with the elevated ideas of those whom they desire to please; or they must at least shew a strong wish to excel in literature and science, and repair the loss which they have suffered in a neglected education. How animating is an influence like this!—Yet let not the sluggish or the ignorant in both sexes despair:

> From Hymen's bands O be not scar'd!
> For blockheads are with blockheads pair'd.

The first female mathematical meeting was held at the house of the London lady, between whom and her promising pupil it had been previously concerted, that, instead of ushering in cards at one of her tea-parties, the eloquent hostess should introduce a little geometrical amusement. This attempt succeeded very well; although the task was rather irksome to the two ladies of exhibiting by themselves a few curious problems and exercises. For not one of the company was qualified to join them: Some of them had not even learnt the common rules of Arithmetic, and others from want of practice found themselves very *rusty* and in no respect qualified to bear a part in the *polished* entertainment of the evening. But a good innovation had, however, been made, which, in an affair of this kind, was of great consequence. The ladies shewed themselves persons of good sense, by appearing gratified with the novelty of the amusement, the utility of which they had also the means of appreciating. Not many weeks had elapsed, when a lady, who had been a guest on this occasion, invited nearly the whole of the former party, with the addition of a few others. The same laudable method of amusing the company was adopted; and the numbers were increased of those who could unite in the fascinating employment. Indeed nearly all of them were in a course of instruction to qualify themselves, though some of them would not venture to display their mathematical prowess till they had attained a little more confidence in their own abilities.

Since that period many have been the friendly parties of learned ladies, and an unceasing fund of mental entertainment have they afforded to each other. Matters were so judiciously managed at first, as to admit no men into these meetings; because they were afraid that their own slender acquirements would be a fit subject of ridicule to many of their male friends who were daily conversant with figures and accounts. But this dread has gradually subsided; as, on becoming better acquainted, their natural shrewdness has convinced them, that in a trial of skill they are able to cope with some men who are accounted proficients in Mathematical learning, and that they are much superior to others who know nothing beyond the common rules of Arithmetic.

Being myself a lover of the Mathematics, I was lately invited to a tea-party of this description. I approached the dwelling of the respectable gentleman with much diffidence, and found thirteen ladies and four gentlemen in the parlour when I arrived. We were served with tea, and enjoyed some excellent social converse during the repast and for upwards of an hour afterwards. We confined ourselves to no particular subject, but discoursed about improvements in the roads, badness of trade, the state of parties, the progress of religion, the spread of knowledge, and other interesting topics. A beautiful little boy then brought a number of tablets to his mother, who distributed them among the company. "Tablets" I discovered to be only another name for little slates with frames of red leather. We then divided ourselves into separate parties, consisting of four or five persons each, and occupied different tables. We had three or four sets of Select Amusements in Philosophy and Mathematics, Bonnycastle's Mensuration, Simson's Euclid, the Leeds Correspondent, the Ladies' and Gentleman's Diaries, Leybourn's Repository, Simpson's Algebra, Gregory's Elements of Trigonometry, Marrat's Mechanics, Bonnycastle's Astronomy, Vyse, Hutton, and Joyce's Arithmetics, Holiday's Fluxions, Crocker's Land-surveying, and Hutton's Mathematical Tables. These are, I believe, a kind of common property, and are conveyed from one house to another on these interesting occasions. They were as equally distributed among the tables as their numbers would admit. We had likewise, at the largest table, one copy of Hutton's Mathematical and Philosophical Dictionary, his Course of Mathematics, Vince's System of Astronomy, Robertson's Navigation, Sanderson's and Euler's Algebra, and Gregory's Mechanics. These were used as books of occasional reference. The persons at each table elected their own president, and I was flattered by being appointed supervisor of the whole concern. If any difficulty occurred and the president of that table was not able to solve it, application was made to me and I endeavoured "to shew cause" and to unravel mysteries. No questions or problems were delivered in form *ex cathedra;* but every one, both the president and his assessors, took up the book best fitted to his or her powers, and derived amusement from the working of some arithmetical question, or the solution of a problem in a higher order of the Mathematics. This was handed to the president of the table for his or her approval. In some of these parties, when all at one table were nearly equal in their attainments, it was the practice for every one to give out a question in his or her turn and all the rest endeavoured to answer it, till each of them had delivered the mathematical problem which was considered the most puzzling.

Such was the diversity of their operations! Yet it was constantly enlivened by agreeable conversation and innocent raillery; and I could not avoid overlooking the assemblage with a degree of satisfaction, which, as a mathematician, I am unable to describe. Part of my mental soliloquy amounted to these reflections: "I am now surrounded by a company of immortal beings, who are partaking of a rational entertainment, the tendency of which is the ennobling of their faculties and the improvement of their minds. It is evident from the cheerfulness of their looks, and the vivacity of their converse, that they are delighted "with their employment. Had not a master-spirit "chalked out this diversion for them, it is probable "that they would on this occasion have been engross"ed in the intricate *doctrine of chances* by means of "cards. The features now so animated and intelli"gent, must then have been beclouded with care and "anxiety. In those games the temporary success of "one is generally embittered by the consideration, "that all the advantages so quickly obtained may be "as speedily lost. And the disaster of another, even "when no money is at stake, imparts to the individual "a deep tinge of disappointment and tends greatly to "depression. The smiles and pleasantry of the mo"ment may strive to hide the disquietude and peev"ishness which rankle in the bosom; but nature will "often break out from beneath these slight *coverings* "and display herself in hasty imprecations and a fret"ful countenance.

"But in the company now before me, no corroding "affections allied to these can be felt at any time: "For among the individuals that occasionally strive "together, even those who cannot produce a correct "answer derive a considerable benefit, and add to the "stores of their knowledge, by observing the labours "of others. Their emulation is likewise excited and "their taste improved, by beholding the various ways "in which proper solutions may be produced, and by "choosing out of them those which are the most con"cise and elegant. The diversified occupations in "which they are now engaged, seem, in another view, "of much importance: They are preparatives for "higher acquisitions. They lay the most stable basis "for all sound reasoning. The real principles of mo"ral argumentation, and the laws of evidence in their "greatest perfection, are deducible from the Mathema"tics. It is not for the sole purpose of entertaining "each other in a satisfactory manner while in a com"pany like this, that these pursuits are deserving of commendation; they are worthy of attention on account of the information and amusement which they yield in private. Many hours may be profitably spent in exercises of this description, when other duties have no claim on the time; and the progress of children in these necessary branches of education, may be much better appreciated when the parents are themselves acquainted with the principles.—Here is none of the stiff formality observable at card-tables. At those blocks of heart-burning and insipid contention, one must continue to run one dull round of vague *Profit and Loss.* And when any variation in the amusement is desired, it proves to be only a road more sinuous in its windings, along which fashion drags "its restless jaded fools." But here every one is at liberty to choose his own diversion; and whatever that may be, it has utility for its object and leaves some profit behind."

These were some of my reflections at the time. We were engaged upwards of two hours in mathematical recreations, and afterwards spent an hour in instructive discourse; during which it was more than obliquely hinted, that the ladies especially were under weighty obligations to the good woman "of famous London town," for having brought into vogue such a happy method of spending a winter's evening. I must add that these accomplished females gave themselves no consequential airs, neither are they dreaded in other circles for being a little more learned than the rest of their sex. The former circumstance redounds highly to their own credit and to the honour of mathematic lore; the latter is reputable to the age in which we live. I am unaccustomed to what is commonly called "polite society;" but, though the company consisted of persons that had ascended to a greater height than myself in worldly distinction, I felt no embarrassment at all after I had been a few minutes in the room, and I departed from the house much delighted with this novel species of entertainment. In my walk homewards the consequences that might accrue, furnished me with sufficient matter of pleasing anticipation; and I arrived at my own door much sooner than I expected.

At a future opportunity I may be inclined to give you some further information on this subject. But having thus practically shewn the superiority which mathematical amusements have over cards and other unmeaning methods of killing time, allow me to conclude by apologizing for the length of this communication, and to assure you that in my other papers BREVITY shall be a part of my study.

Mytholme, DIDASCALUS.

Is "Than" a Preposition?

To the Editor of the Leeds Correspondent.

A variation in the practice of polite writers, occurs in the use of the word *than*. Some of them, who have enjoyed the benefits of a classical education, consider it, like common people, sometimes as a conjunction; but, on other occasions, they employ it as a preposition governing the objective case. There is no dispute respecting the word *than* when it is employed in its conjunctive capacity, as in the following sentences: *Would you ruin me rather than him?—It was more worthily bestowed on him than thee.* In the first of these instances, it couples the similar ~~cases~~ of the two pronouns; and, in the second, it is generally said to act the same part.

But the contest relates to its influence as a preposition. It is used in this form when we say, *Was he any better served than me? It was no higher personage than him.* The persons that have had only a common English education, not being aware of any grammatical reasons for this construction, call it in both examples "erroneous;" and contend that the ellipsis, when properly supplied, is, *Was he any better served than I* *was? It was no higher personage than he was.* The learned, on the contrary, argue that the word *than* has an intrinsic force sufficient to govern the following substantive or pronoun in the objective case; and that the practice of all our ancient authors, and of the best among the moderns, is uniformly in favour of this usage.

Without arrogating to myself the province of arbitrator between the disputants, I have met with an expression which cannot be accounted for by the common English expedient of supplying the ellipsis. It is this, *He shewed himself to be a nobler man than thee.* It may indeed be explained by this rule, "The substantive verb *to be*, and all passive verbs, require the same case after them which they have before them."

I wish to know whether this mode of expression be an indigenous one, or borrowed from the Latin, in which tongue the learners are early shewn that "*In, with, through, for, from, by,* and THAN after the Comparative Degree are signs of the Ablative Case." If it be proved by authentic quotations, that the word *than* in English possesses this governing property, may not the following QUERY be submitted to the consideration of Grammarians? "Is it *than*, or the *comparative degree,* that exercises an influence over the substantive or pronoun that follows it in the objective case?"

TYRO.

HONOURS DUE TO LEARNING.

ONE day the emperor Charles the Fifth, being informed that *Guicciardini,* the great Italian historian, attended his levee, gave immediate orders for him to be admitted into his dressing-room, and conversed with him on the subject of the history in which he was engaged. In the mean time, one of the courtiers informed his majesty, that a murmur was excited among several persons of quality and officers of the army, who for many days together had been desiring an audience without being allowed that honour. The emperor, holding *Guicciardini* by the hand, entered the drawing-room, and thus addressed the company. "Gentle-"men! I am informed you think it strange that I "gave admission to *Guicciardini* before you; I desire "you, however, to consider, that in one hour I can "create a hundred nobles, and a like number of officers "in the army, but I cannot produce such a historian "in the space of twenty years. What benefit is there "in your solicitude to execute your respective func-"tions in the camp or in the council, if historians, "for the instruction of your descendants, do not trans-"mit an account of your conduct to posterity? Who "are they who have informed mankind of the heroic "actions of your ancestors, but historians? It is neces-"sary then to honour them, that they may be encou-"raged to convey your illustrious deeds to futurity. "Thus, gentlemen, you ought neither to be offended "nor surprised at my regard for *Guicciardini*, since "you have as much interest in his province as "myself."

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REMARKS ON THE IMPERATIVE MOOD.

To the Editor.

Sir,—A short remark which I made on the Imperative Mood having been honoured with a place in your very useful and comprehensive Miscellany, I feel encouraged to submit to your inspection one or two other observations on that subject. It has been before observed that the Imperative Mood is, in philosophical strictness, entitled to one person only, that is, to the second, on account of the uniformity of its address to that person. But the general analogy of languages ancient and modern, in which some admit two persons and others three, carries so much authority with it, that it seems to form an unconquerable bar against any alteration or improvement in this part of our Grammar. On this account alone it is that we find a positive uniformity among all our Grammarians; and but few of them have attempted to mention any deviation from the general form, or come to any explication whatever. It is evident, however, that the present construction of the Imperative is composed of two verbs, both an Imperative and an Infinitive; and I can see no rational way of analyzing such a clause of a sentence as *Let me rule,* but by considering *let* the Imperative, and *Rule* the Infinitive Mood.

Since I wrote my first note upon this subject I have witnessed an objection to this principle, supported by quotations from the *Lord's Prayer,* &c. The objection is, "Do not the following sentences prove that the Imperative has a third person, '*Hallowed be thy name! Thy Kingdom come! Thy will be done?*'" To this it should be answered, that the expressions *Be hallowed, come,* and *Be done,* are not in reality imperative, but Potential verbs; and the construction is, *May thy name be hallowed! May thy kingdom come! May thy will be done!* But if it were contended that the construction is, *Let thy kingdom come!* &c., it would but afford a still stronger proof that the Imperative has claim to one person only. For if we suppose the verb *let* to be understood, it is evident it is the second person agreeing with its nominative *thou* implied, and accordingly governs *come* in the Infinitive Mood. Another quotation similar to this I shall here introduce from Mr. Pope's *Universal Prayer,* which may serve to illustrate the point.

> This day be bread and peace my lot,
>   All else beneath the sun,
> Thou know'st if best bestow'd or not,
>   And let thy will be done.

It is easy to perceive that *be* in the first line ought to be parsed a verb in the Potential Mood, as though it were transposed *May bread and peace be my lot!* and in the last line *let* is the Imperative, &c. and *be done* the Infinitive Mood.

It does appear then that the Imperative in English admits of no more than one person; and how far we ought to follow the peculiarities of other languages remains to be determined. With respect to the Latin and Greek, they do with propriety admit of a third person, because they have a distinct and appropriate variation from the second: thus, when we say *amato* and *amanto* in Latin, we have no corresponding translation for them in English without making use of the active verb *let*, a personal pronoun, and an infinitive verb; as *let him love, let them love.*

The same might be said of the Greek, which, however, varies from the Latin in not having a first person plural falsely imputed. But every classical scholar knows that neither of these languages is strictly entitled to a first person Imperative: for though grammarians allow it in the plural in Latin, it is certainly always the same as the first person plural in the Potential, and ought to be so applied.

I was led into this subject by the daily exercise which my present avocation calls me to perform; and have invariably found it necessary to proceed upon the plan of analysis just laid down. I am well aware, Sir, that without a constant (and, I had almost said, a daily) attention to parsing, it is impossible to lead the young student to a competent knowledge of any language; and every attempt to simplify its principles ought to be seriously regarded by those who are employed in the important office of instructing the rising generation. I should be glad to know the opinion of your "Correspondents" on the subject.

JOSEPH HILL.

Mr. Richardson's Academy,
Market Weighton.

ANSWERS

TO THE

QUESTIONS FOR YOUTH,

Proposed in the Fourth Number of the Second Volume.

(1) *By* Master JOHN WILLIAMS.

To find three numbers in geometrical progression, (by a simple equation) whose sum shall be 7, and the sum of their squares 21.

Answered by Master W. VINT, *Idle.*

Let $x + y + z = 7$

And $x^2 + y^2 + z^2 = 21$

$$x : y :: y : z$$
$$xz = y^2$$
$$x^2 + xz + z^2 = 21$$
$$x^2 + 2xz + z^2 = y^2 + 21$$
$$x + z = \sqrt{y^2 + 21}$$
$$x + z = 7 - y$$
$$7 - y = \sqrt{y^2 + 21}$$
$$49 - 14y + y^2 = y^2 + 21$$
$$49 - 14y = 21$$
$$14y = 49 - 21 = 28$$
$$y = 2$$
$$x + z = 7 - 2 = 5$$
$$x^2 + z^2 = 21 - 4 = 17$$
$$xz = y^2 = 4$$
$$2xz = 8$$
$$x^2 - 8 + z^2 = 9$$
$$x - z = 3$$
$$x + z = 5.$$

Then $x = 4$ and $z = 1$.

Again by Master C. **Bartholomew**, *Woodhouse Grove.*

Let x, y, and z denote the numbers. Then per question $x + y + z = 7$. And $x^2 + y^2 + z^2 = 21$.

From the square of the first equation take the second, then dividing the remainder by 2 we have $xy + xz + yz = 14$. Or since $xz = y^2$ by the nature of geometrical progression we have $xy + y^2 + yz = 14$. Then by division $y = \frac{14}{x+y+z} = \frac{14}{7} = 2$. Hence $xz = y^2 = 4$ and $x + z = 7 - y = 5$. Then from the square of this sum taking 4 times the product, there results $x^2 - 2xz + z^2 = 9$. Or by extracting the root $x - z = 3$. But $x + z = 5$. Therefore by addition $2x = 8$, or $x = 4$. Whence $z = \frac{y^2}{x} = \frac{4}{4} = 1$, and the numbers required are 1, 2, and 4.

(2) *By* Master C. Saunders.

If a square, the side of which is 10 inches, be made to revolve about its diagonal, required the content of the solid described in one revolution.

Answered by Masters B. Schwartzenbrook, and G. Aspden.

First $10\sqrt{2} = 14.1421 =$ the diag. of the square or diam. of the base of each cone. Hence $14.1421^2 \times 14.1421 \times .2618 = 200 \times 10\sqrt{2} \times .2618 = 523.6\sqrt{2} = 740.48035 =$ the solidity required.

(3) *By* Master F. Gaskin.

$$\text{Given}\left\{\begin{array}{l} x - 9y - 99z = 576 \\ x - y^2 - z^3 = 56 \\ x - 14y - z^3 = 11 \end{array}\right\}\text{to find } x \text{ the year, } y \text{ the month, and } z \text{ the day, of my birth.}$$

Answered by Master ——— Wigg.

By taking the second equation from the third $y^2 - 14y = -45$, from which $y = 5$ or 9. Again by taking the second from the first $y^2 - 9y + z^3 - 99z = 520$, by writing $y = 5$, we have $z^3 - 99z = 540$, reduced $z = 12$, therefore $x = 1809$, then the proposer was aged 10 years on May 12th, 1819.

Again by Master Charles Macauley, *Liverpool.*

From the third equation take the second and we shall have $y^2 - 14y = -45$, whence $y = 7 \pm 2 = 5$ in this case. Substitute this value of y in the first and second equations

$$\begin{aligned} x - 99z &= 576 + 45 = 621 \\ x - z^3 &= 11 + 70 = 81 \\ \text{whence } z^3 - 99z &= 540 \text{ and } z = 12 \end{aligned}$$

pr. first $x = 576 + 1188 + 46 = 1809$ the year; therefore Master Gaskin was born May 12, 1809, two years after I first saw the light.

(4) *By* Master J. Hodge.

$$\text{Given}\left\{\begin{array}{l} x^3 - 6x^2y = 72y^3 - 12xy^2 \\ n^4y - \frac{1}{3}n^2xy = \frac{1}{9}x^3 \end{array}\right\} \text{to find } x \text{ and } y \text{ in terms of } n^2 \text{ by a simple equation.}$$

Answered by Master W. Vint.

By transposing the first equation $x^3 - 6x^2y + 12xy^2 - 8y^3 = 64y^3$, or $x - 2y = 4y$, $x = 6y$, by substituting $6y$ for x in 2nd equat. and dividing by y, $n^4 - 2n^2y = 24y^2$, add y^2 to both sides and $n^4 - 2n^2y + y^2 = 25y^2$, or $n^2 - y = 5y$, $6y = n^2$ or $y = \frac{n^2}{6}$, and $x = 6y = n^2$.

Again by Master —— Wigg, *Cambridge.*

By transposing and subtracting $8y^3$ from each side $x^3 - 6x^2y + 12xy^2 - 8y^3 = 64y^3$ by evolution $x - 2y = 4y$, then $x = 6y$, by writing this value of x in the second equation, $n^4y - 2n^2y^2 = 24y^3$ or $24y^2 + 2n^2y = n^4$ or $y^2 + \frac{1}{12}n^2y = \frac{n^4}{24}$ reduced $y = \frac{1}{6}n^2$ therefore $x = n^2$.

(5) *By* Master G. GREGORY.

Given two sides of a plane triangle = 36 and 25, and their included angle = 80°, to find the third side and the radius of the circumscribing circle.

Answered by Masters JOHN HARE *and* J. LOCUS.

By Trig. as $36 + 25 : 36 - 25 :: \text{tang} \frac{1}{2}(180° - 80°) = 50° : \text{tang.}\ 12° 7' 40'' = \frac{1}{2}$ the diff. of the unknown angles; hence the greater angle is 62° 7′ 40″, and the less 37° 52′ 20″; again as Sine 37° 52′ 20″ : 2.5 :: sine 80° : 40.105 = the third side of the triangle; and as radius : 36 :: Sine 37° 52′ 20″ : 22.101 = the perpendicular. Lastly by Euclid, prop. c. book vi, $\frac{1}{2}((36 \times 35) \div 22.101) = 20.361$ = the radius of the circumscribing circle.

Again by Master ROBERT TAYLOR.

Here $36 + 25 : 36 - 25 ::$ tan. 50° ½ sum opp. $\angle$s : 12° 8′ tang. ½ diff. Then 50° + 12° 8′ the greater $\angle$. And s.62° 8′ : 36 :: s80° : 40.1 the third side. Also s80° : 20.05 :: 90° : 20.36 the radius of the circumscribing circle.

(6) *By* Master R. WEST.

Required the side of that cube whose solidity subtracted from the side itself shall leave the greatest remainder possible.

Answered by Master ROBERT TAYLOR, *Plymstock.*

Put x = side of the cube, then x^3 is its solidity and $x - x^3$ is to be a max. by the question: in fluxions $x' - 3x^2x = 0$ this divided by x' and reduced gives $x = \sqrt{\frac{1}{3}} = \frac{1}{3}\sqrt{3} = .5773502$.

Again by Master J. LOCUS.

Put x = the side, then $x - x^3$ = a max. which in fluxions is $x' - 3x^2x' = 0$; dividing by x', &c. gives $x = \sqrt{\frac{1}{3}} = \frac{1}{3}\sqrt{3} = .57735026$, the answer.

(7) *By* Master W. Godward.

Measuring a field in form of a right angled triangle, I found the base AB = 1820 links; but as I could neither measure the perpendicular BC, nor the upper part of the hypothenuse, by reason of obstructions, I took a station at D 1000 links from the right angle; from this station I let fall a perpendicular on the hypothenuse AC and found it = 730 links. Required the area of the field.

Answered by Master Drake Wardwel.

$\sqrt{820^2 - 730^2} = 473.496988$ lower part of the hypothenuse. Then by theo. 4, book 6 Euc. as 373.496988 : 1820 :: 730 : 3557.190667 perpendicular. And (3557.190667 × 1820) ÷ 2 = 3237043.50997 links = 32 acres, 1 rood, 19 perches. Answer.

Again by Master J. Locus,

First $\sqrt{820^2 - 730^2} = 10\sqrt{1395} = 373.496988$ = the lower part of the hypothenuse, and by similar triangles (vide Prop. 4, book vi, Euc. Elem.) 373.496988 : 1820 :: 730 : 3557.19066 = the perpendicular. Hence (3557.19066 × 1820) ÷ 2 = 3237043.5 square links = 32 acres, 1 rood, $19\frac{1}{4}$ poles, the answer.

(8) *By* Master J. Roberts.

The angle at the vertex of a right cone is $38\frac{1}{2}$°, and perpendicular height 5 feet. Whereabouts must a section parallel to the base be made, that the area of the plane of that section may be to the convex surface of the whole cone as 1 : 8? Also required the distance of the centre of gravity of the cone cut off, from the centre of gravity of the remaining frustum.

Answered by Master J. Locus.

As the radius : 5 :: tang. 19° 15′ (half the vertical angle) : 1.74603 = the radius of the cone's base; therefore the diameter is $1.74609 \times 2 = 3.49212$, the slant side $\sqrt{5^2 + 1.7406^2} = 5.2961$, and the convex surface $(3.49212 \times 3.1416 \times 5.2961) \div 2 = 29.05134395$; hence the area of section is $29.05134395 \div 8 = 3.63166799$, and the diameter $\sqrt{3.63166799 \div .7854} = 2.15034$. Again by similar triangles $3.49212 : 5 :: 2.15034 : 3.0216$ = the height of the cone cut off, and consequently the height of the frustum is $5 - 3.0216 = 1.9784$. Lastly, by Marrat's Mechanics, Art. 656.

$$\frac{3 \times 1.74606^2 + 2 \times 1.74606 \times 1.07517 + 1.07517^2}{1.74606^2 + 1.74606 \times 1.07517 + 1.07517^2} \times \frac{19784}{4} = 1.14312$$

= the distance of the centre of gravity of the frustum from the less end, and the distance of that of the cone from the base is $\frac{3.0216}{4} = .7554$, hence $.7554 + 1.14312 = 1.89852$ = the distance of the centre of gravity of the cone cut off from the centre of gravity of the remaining frustum.

Ingenious solutions to all the questions were given by Masters C. Macauley, *Hope School, Liverpool;* Wm. Vint, *son of the Rev. Wm. Vint, Idle;* Wm. Godward, *Free Writing School, Sheffield;* John Gillott, *Lancasterian School, Sheffield;* and J. Locus, *Nottingham.*

Masters Robert Taylor, *West House School, Plymstock, near Plymouth,* and Drake Wardwel, *Chesterfield,* answered 6 questions.

Master Geo. Aspden answered 4 questions, and Master Bailey Schwartzenbrook, *Mr. H. Lee's Academy, Windhill, near Bradford,* answered 2 questions.

Master Samuel Walker, *pupil in Mr. S. Littlewood and Son's Academy, Townhead, Rochdale,* answered 4 questions.

Master John Hare, *Woodhouse Grove Academy,* answered 3 questions, and Master Samuel Waddy, *at the same Academy,* answered the 7th question.

ANSWERS
TO THE
MATHEMATICAL QUESTIONS

Proposed in the Fourth Number of the Second Volume.

(163) *By* E. T. *a new correspondent.*

The roots of the equation $x^3 - px^2 + qx - r = 0$, are a, b, c: required the equation whose roots are $(a+b,)$ $(a+c,)$ $(b+c.)$

Answered by the PROPOSER, and Messrs. W. GODWARD, and J. GILLOTT.

Now in the given equation $p = a + b + c$,
$q = ab + ac + bc$,
$r = abc$.

Let P, Q, R be the corresponding co-efficients of the transformed equation.

Then $P = 2.\overline{a+b+c} = 2p$.
$Q = a^2 + b^2 + c^2 + 3.\overline{ab+ac+bc} = p^2 + q$.
$R = \overline{a+b}.\overline{c+a}.\overline{b+c} = a.\overline{ab+ac+bc} + b.\overline{ab+ac+bc} + c.\overline{ac+cb} = \overline{a+b}.q + c.\overline{q-ab} = \overline{a+b+c}.q - abc = pq - r$
= the last term of the transformed equation ∴ the required equation will be $x^3 - 2px^2 + (p^2 + q).x - (pq - r) = 0$.

Messrs. Baines, Garside and Whitehead also answered this question.

(164) *By* G. MALING, Esq. *Scarbro'.*

If $\sqrt[3]{(a+x)} + \sqrt[3]{(a-x)} = b$, then will $x = \sqrt{a^2 - \left(\frac{b^3 - 2a}{3b}\right)^3}$ Required proof.

Answered by Messrs. J. BAINES and J. HARKER.

Put $a + x = m^3$, and $a - x = n^3$; then $m + n = b$, and $m^3 + n^3 = 2a$. From the cube of the first equation take the second, and $3mn.(m + n) = b^3 - 2a$; but $mn = \sqrt[3]{m^3n^3} = \sqrt[3]{(a+x).(a-x)} = \sqrt[3]{a^2 - x^2}$, and m

$+ n = b, \therefore \sqrt[3]{a^2 - x^2} = \frac{b^3 - 2a}{3b}$, or $x = \sqrt{\left\{a^2 - \left(\frac{b^3 - 2a}{3b}\right)^3\right\}}$.

(165) *By* Mr. T. S. Davies, *Sheffield.*

Given $\sqrt[9]{(16 + x)} + \sqrt[9]{(16 - x)} = 2$, to find x.

Answered by Messrs. W. Godward and J. Gillott.

Put $16 + x = m^9$, and $16 - x = n^9$; then $m + n = 2$. and $m^9 + n^9 = 32$. Now $m^3 + n^3 = (m + n)^3 - 3mn\,(m + n) = 8 - 6mn$, and $m^9 + n^9 = (m^3 + n^3)^3 - 3m^3n^3\,(m^3 + n^3) = (8 - 6mn)^3 - 3m^3n^3\,(8 - 6mn) = 18m^4n^4 - 240m^3n^3 + 864m^2n^2 - 1152mn + 512 = 32$, (by the quest.) or $m^4n^4 - \frac{40}{3}m^3n^3 + 48m^2n^2 - 64mn + \frac{80}{3} = 0$; but this equation is divisible by $m^2n^2 - 4mn + 4 = (mn - 2)^2$, the quotient being $m^2n^2 - \frac{28}{3}mn + \frac{20}{3} = 0$; hence $mn = \frac{14 - 2\sqrt{34}}{3}$ but $mn = \sqrt[9]{m^9n^9} = \sqrt[9]{(16 + x).(16 - x} = \sqrt[9]{256 - x^2}, \therefore \frac{14 - 2\sqrt{34}}{3} = \sqrt[9]{256 - x^2}$, or $x = \sqrt{256 - \left(\frac{14 - 2\sqrt{34}}{3}\right)^9} = 15.99668$, the answer.

Answers were also received from Messrs. Baines, Garside, and Whitehead.

(166) *By* Mr. E. Webster.

$$\text{Given}\left\{\begin{array}{l} x^2 + y^2 + xy + xz - yz = \square \\ x^2 + z^2 + xy + yz - xz = \square \\ y^2 + z^2 + yz + xz - xy = \square \\ \quad \text{and } xyz \ldots\ldots\ldots\ldots\ldots\ldots = \square \end{array}\right\}\begin{array}{l}\text{To find} \\ \text{the value} \\ \text{of } x, y, z.\end{array}$$

Answered by Messrs. J. Baines and E. Webster.

Put $x^2 + y^2 + xy + xz - yz = (x + y)^2 = x^2 +$

$2xy + y^2$, and we have $xy = xz - yz$, or $y = \frac{xz}{x + z}$: these values being inserted in the 2nd, 3rd, and 4th formulæ give $x^2 + z^2$, $y^2 + 2yz + z^2$, and $\frac{x^2z^2}{x + z}$ to be made squares, but the 2nd, and the numerator of the the 3rd are squares; hence we have only to make $x^2 + z^2$, and $x + z$ squares, and for this purpose put $x = 3n$, and $z = 4n$, then $x^2 + z^2 = 25n^2$, and $x + z = 7n$, which will be a square when $n = 7$; hence $y = 12$, and the three numbers required 21, 12, and 28, which are probably the least.

It was also answered by Messrs. Garside and Whitehead.

(167) *By* E. T.

Having given the focal Length of a Lens whereby a person may see distinctly at a given distance, to find the focal length of a Lens which will enable him to see distinctly at any other given distance.

Answered by the Proposer.

In order that the person may see both the objects P and Q distinctly, it will be necessary that the rays diverging from the objects, and incident upon the lenses must after reflection converge to some point q. Let E be the place of the lens (here supposed convex), QE the distance at which he can see objects with it, FE its focal length. Let PE be the distance at which he wishes to view objects with another lens whose focal length is Ef. Put QE $= a$, PE $= b$, FE $= f$, fE $=$ x. Then QF : QE :: QE : Qq $= \frac{\text{QE}^2}{\text{QF}} = \frac{a^2}{a - f}$; and similarly P$q$ $= \frac{\text{PE}^2}{\text{P}f} = \frac{b^2}{b - x}$. Now since QF : QE :: QE : Qq, therefore FE : QE :: Eq : Qq, and QE : PE :: $a : b$, also Pf : PE :: PE : Pq, therefore *div. et inv.* PE : Ef :: Pq : Qq; whence *componendo* FE : fE :: $a \times$

Pq : $b \times$ Qq, that is $f : x :: \frac{ab^2}{b-x} : \frac{a^2b}{a-f}$: whence $x = \frac{abf}{ab-bf+af}$ = focal length required.

It was also answered by Mr. Garside.

(168) *By* Mr. J. Baines, *Nottingham.*

If any two straight lines whatever be drawn from the angles at the base of a plane triangle intersecting each other in the perpendicular, drawn from the vertical angle to the base, and meeting the opposite sides in two points, the lines joining these points and the foot of the said perpendicular shall make equal angles with the base. Required a demonstration.

Answered by Amicus.

Demonstration. Let AH and BL, any two straight lines drawn from the angles A, B, of the plane △ ACB, intersect in CD the perpendicular to AB, and meet the opposite sides BC, AC in H and L: LD and HD joined make equal angles with the base AB.

Draw LE and HF perpendicular to AB, and through L, H, draw LP intersecting CD in I, and AB produced in P. Then by question 145, No. 13, Math. Companion, or question 14 Ladies' Diary, 1818, AP is harmonically divided in D and B, and consequently by the property of harmonicals LP is also harmonically divided in the points I and H, that is LI : HI :: LP : HP :: (by parallel lines) ED : DF :: EL : FH ∴ the right angled triangles DEL and DFH having the sides about the right angles proportionals, are equi-angular, and consequently the angles ADL and BDH are equal. Q. E. D.

Cor. 1. CD bisects the angle LDH.

Cor. 2. Hence if AH be perpendicular to BC, it will bisect the angle DHL.

Cor. 3. If AH and CD be perpendicular to BC and AB, the point of intersection M will be the centre of the circle inscribed in the triangle DHL.

Again by Mr. J. Whitehead.

Let the figure be drawn as per above, and draw GMN par. to AB: then by sim. triangles we shall have the following proportions, viz.

$$\begin{array}{r}
MQ = DF : AF :: MN : AB \\
MN : BD :: CM : CD \\
AF : FH :: AD : DM \\
AD : GM :: CD : CM \\
BD : DM :: BE : LE \\
GM : AB :: DE : BE \\
\hline
\end{array}$$

Or *DF : FH :: DE : LE By mult.

*Hence as the triangles DEL, DFH have the sides about the right ∠s proportional, they are similar and consequently ∠ ADL = ∠ BDH. Q. E. D.

Messrs. Garside, Lightbown, and Swale answered this question.

(169) *By* Mr. J. Baines, *Nottingham.*

With what velocity must a cylindrical vessel full of water and standing perpendicular to the horizon, be whirled round its axis, so as to cause by its rotation as much water to be thrown out as will make the centre of its base appear dry? The diameter of the cylinder being 4 and its altitude 7 feet.

Answered by Mr. H. Lightbown.

Let ABDC represent a section of the cylindrical vessel, EF its axis, AVB the curve surface of the water, formed by the circular motion about the axis EF; draw the ordinate GH parallel to AB.

Put EB $= 2$ feet $= r$, EF $= 7 = h$, $16\frac{1}{12} = g$, GV $= x$, GH $= y$, the velocity at the distance r from the axis EF, $= v$, $.7854 = n$, and the quantity of water thrown out, or the solidity of the hollow part of the cylinder AVHB, caused by the whirling motion round the axis, with the velocity v, $= q$.

Now the hollow part of the cylinder of water AVHB, is the conic paraboloid, whose content (q) is well known to be one half of its circumscribing cylinder ABQR, and the equation of the curve is $4r^2gx \div v^2 = y^2$, (see Mr. Cunliffe's solution to question 193, page 163 of the new series of Leybourn's Repository); but the general value of x is $= \frac{q}{2nr^2}$; therefore, when $y = r$, and $x = \frac{q}{2nr^2}$, in the equation of the curve, $v^2 = 4g \times \frac{q}{2nr^2} = \frac{2gq}{nr^2}$, or $v = \sqrt{\left(\frac{2gq}{nr^2}\right)}$. In the present case, when QR coincides with CD, V will be the centre of the base, and therefore $q = 2nr^2 \times h$; or $v = \sqrt{4gh} = 2\sqrt{gh} = 2\sqrt{16\frac{1}{12} \times 7} = 21.22$ feet per second the velocity required.

When $q = 0$; then x, and v are each $= 0$.

When $q = 4nr^2h$, or that the whole quantity of water is thrown out of the cylinder; then $v = \sqrt{8gh} = 2\sqrt{2gh} = 30.10$ feet the velocity in that case.

The Proposer also answered it.

(170) *By* Mr. J. Garside, *Mirfield.*

The axletree of a certain mill wheel makes just 100 revolutions in a minute, and at the end of it there is a wheel of 105 teeth, which turns another wheel of 15 teeth. Now the wheel of 15 teeth, and a large grinding stone weighing 12 cwt. are both fixed on the same axle. I desire to know how many times the weight of the stone acts on its own axis: the chord of 16° of the stone being 8 inches.

Answered by Mr. C. Waterhouse, *Eccleshill.*

From the data the diameter of the grinding stone is readily found $= 4.79$ feet. Put $\frac{2}{3}$ of 4.79 or 3.19$\frac{1}{3}$ feet $= d$, the diameter of the circle of percussion, $t = 60$

* * *

$\div 700 = \frac{3}{35}$ of a second, the time of one revolution, $q = 3.1416$, and $s = 16\frac{1}{12}$, then by Banks's Lectures, page 3rd, $F = \frac{d \cdot q^2}{s \cdot t^2} \times w = 266.73$ times the weight of the stone, as required.

Again by the Proposer.

By the question it is evident that the grinding stone makes 700 revolutions in one minute because the wheel of 15 teeth turns seven times as often as the wheel of 105 teeth. Let ABC represent a section of the stone, draw the perpendicular BD to AC and CE perp. to AB. By Trigonometry rad. 1 : side 8 :: sine 8° : 1.1133848 = AD. And by similar triangles ADB, AEC 1.1133848 : 8 :: 4 : 28.7411863 = BC the radius, which put = r, hence the velocity at the circumference of the stone is 3.1416 × 55.88564 feet in a second, which put = v also put $2g = \frac{193}{6}$ = gravity, p = 3.1416, the weight of the stone = w, its thickness = t, any variable radius of the stone = x and $m = px^2t$ = the solidity when the radius is x: then will the velocity at the circumference at the distance x from the centre $= \frac{vx}{r}$ and the centrifugal force at that distance is $\frac{v^2x}{r^2}$. Now $\frac{v^2x}{r^2} \times m \times \frac{1}{2g} = \frac{pv^2t}{2gr^2} \times 2x^2x'$ its fluent is $\frac{pv^2t}{2gr^2} \times \frac{2}{3}x^3 = \frac{v^2x}{2gr^2} \times \frac{2}{3}m$ which when x becomes $= r$ gives $\frac{v^2rm}{3gr^2} = \frac{v^2m}{3gr}$, or expounding m by w the weight of the stone, we have $\frac{v^2w}{3gr} \doteq 266.7359\,w$, that is 266.7359 times the weight of the stone acts on its axis.

(171) *By* Mr. R. MAFFETT, *Plymouth.*

Given the ratio of the sides, the difference of the angles at the base, and the radius of the inscribed circle, to construct the triangle.

Answered by Mr. J. H. SWALE, *Liverpool.*

COMP. Perpendicular to the indefinite line MI, take $IQ = IE$ the given radius, make $\angle IQD =$ half the given one, and $DF : DQ =$ the given ratio: a circle through E, Q, F, determines A, one of the angular points of the required $\triangle$.

Draw AQ, AC, CB, making $\angle QAC = \angle QAD$, and $\angle DCB = \angle DCA$; we have to shew that ACB is the required $\triangle$.

Draw AE, AF, BQ: the $\angle$s BAC, BCA, of the $\triangle$ ACB, being bisected by the lines AQ, CQ, it follows that $\angle ABC$ is bisected by BQ, or Q the centre of the inscribed circle, and $QI =$ the given radius per Comp. Also, as CD bisects the $\angle ACB$, and QI is perpendicular to AB, we shall have $\angle ABC - \angle BAC = 2\angle IQD =$ the given $\angle$: It only remains to prove that $AC : CB =$ the given ratio.

Obviously, $\angle BAQ + \angle DQI = \angle ABQ$, $\angle BAQ = \angle BAE$, and $\angle EAF = \angle EQF$ or $\angle DQI$; therefore, $\angle DAF = \angle DBQ$, or AF par. to BQ, consequently, $DA : DB = DF : DQ = AC : CB =$ the given ratio.

The Comp. will be very little different, if, instead of the radius of the inscribed circle, we have given a line QM drawn from the centre of that circle, to make a given $\angle QMB$ with the base. In this case, draw $MQ = ME$ the given line, making $\angle DMQ = \angle DME$ the given angle, join QE, make $\angle EQF =$ half the given diff. of the $\angle$s at the base, the rest as above.

A great variety of Comps. may be given to this simple prob. Having determined the position of QD as before, draw $QG = 2DI$ and par: to DM, make KQ

: QG = the given ratio, join IK: a circle through Q, G, to contain half the diff. of the ∠s at the base, cuts IK at L, and drawing QL it determines A. The Demonstration we leave to exercise the student.

The prob. under this form, is, to draw, from two given points, two right lines, including a given angle, to cut off segments, from a right line given in position, that shall have a given ratio, a case of the *sectio rationis*.

Again by Mr. Elias Webster.

Const. Make the angle ACD equal to the given difference of the angles at the base, and take AC to CD in the given ratio of the sides: join AD, producing it to meet CB = CD in B: then inscribe a circle in the triangle ACB: and from the centre Q with the radius QL equal to the given radius describe a circle, to which draw the tangents FK, FG, GK respectively parallel to AB, AC, BC intersecting at F, G, K; so shall FGK be the required △.

The Demonstration follows so obviously from the Construction that, for the sake of brevity, we may omit it.

Messrs. Baines, Garside, Lightbown and Whitehead sent good answers.

(172) *By* Ambulator, *Canterbury.*

Given the base and vertical angle of a plane triangle to construct it when the rectangle of the radius of the inscribed circle and a given line are equal to the square of the sum of the sides.

Answered by Messrs. Wm. Stringer *and* Johnson.

Suppose that ACB is the required triangle, DG the diameter of its circumscribing circle bisecting the base AB at right angles in I, DC the line bisecting the vertical angle ACB, and DE perpendicular to AC. In IG take IF equal to the radius of the inscribed circle;

then by well known properties DF = DE and CE = $\frac{1}{2}$ (AC + BC); therefore when the rectangle under IF and a given line is equal to the square of the sum of the sides AC and BC, the rectangle under IF and a given line L will be equal to the square of CE, therefore L is to CE as CE to IF; but because the angle ECD equal to half the vertical angle ACB is given, the ratio of EC to ED or that of EC^2 to ED^2 is given; take 1 to n in this ratio; then compounding this with the preceding analogy and recollecting that DF = DE, we have $nL : DF :: DF : IF$. In IG produced if necessary take IN = nL, and IH = DF, then IN will be a given line, and DI is given because the base and vertical angle are given: hence the last analogy becomes $IN : IH :: IH : IF$; therefore $IN . IF = IH^2$, or by division $HN : IH :: FH = ID : IF :: HN . IH : IH^2 = IN . IF$; wherefore $IH . HN = DI . IN$ = a given rectangle, or the sum IN and the rectangle of the two lines IH, HN are given to determine them, the method of doing which is well known. Hence the construction is obvious.

Mr. J. H. Swale reduces this question to a case of the *sectio rationis*; he also gives the Composition. We are obliged reluctantly, for want of room, to omit it.

The answer received from Mr. Garside and Mr. Stringer are similar.

(173) *By* Mr. R. SMITH, *Manchester.*

At the game of Whist, what are the odds that the dealer has three trumps or more, besides the trump turned up?

Answered by Mr. J. WHITEHEAD, *Huddersfield.*

When the card turned up is taken from the whole pack there remain 51, among which are 12 trumps. Now the dealer must have no trumps, or 1 trump, or 2, or 3, or more trumps; but the probability of having no trump is

$$\frac{39\times38\times37\times36\times35\times34\times33\times32\times31\times30\times29\times28}{51\times50\times49\times48\times47\times46\times45\times44\times43\times42\times41\times40}=$$

$\frac{13\times19\times29\times31\times37}{7\times23\times25\times41\times43\times47}=$ A and that of having one trump $=$ A $\times\frac{12}{28}\times12=\frac{36}{7}$ A; also the probab. of having 2 trumps is A $\times\frac{12}{28}\times\frac{11}{29}\times66=\frac{2178}{7\times29}$ A, therefore A $+\frac{36}{7}$ A $+\frac{2178}{7\times29}$ A $=\frac{3425}{7\times29}$ A $=\frac{137\times13}{23\times41}$ $\frac{\times19\times31\times37}{\times43\times47\times49}=\frac{38813333}{93384347}=$ the prob. of having 0 trump, or 1, or 2 trumps precisely, hence 1 — $\frac{38813333}{93384347}=\frac{54571014}{93384347}=$ the probability of having 3 or more trumps, consequently the odds are as 54571014 to 38813333, or as 27 to 19 nearly.

By the Proposer.

When the trump turned up is taken from the whole pack, viz. 52, there remain 51, of which 12 are trumps, and 39 may be considered as blanks.

In receiving 12 of these 51, the probability of receiving 3 trumps or more may be obtained by seeking severally the prob. of receiving 3 precisely, 4 precisely, and so on to 12 precisely, and adding them together. But instead of this, by seeking severally the probabilities of receiving no trump, one precisely, two precisely, and adding them together, we obtain the probability of the contrary; hence (prob. 6, Simpson's Laws of Chance,) these three probabilities will respectively stand thus:

$$\frac{\frac{39}{1}\times\frac{38}{2}\times\frac{37}{3}\times\frac{36}{4}\times\frac{35}{5}\times\frac{34}{6}\times\frac{33}{7}\times\frac{32}{8}\times\frac{31}{9}\times\frac{30}{10}\times\frac{29}{11}\times\frac{28}{12}}{\frac{51}{1}\times\frac{50}{2}\times\frac{49}{3}\times\frac{48}{4}\times\frac{47}{5}\times\frac{46}{6}\times\frac{45}{7}\times\frac{44}{8}\times\frac{43}{9}\times\frac{42}{10}\times\frac{41}{11}\times\frac{40}{12}}=$$

$\frac{3910797436}{158753389900}$ the probability of none.

$$\frac{\frac{12}{1}\times\frac{39}{1}\times\frac{38}{2}\times\frac{37}{3}\times\frac{36}{4}\times\frac{35}{5}\times\frac{34}{6}\times\frac{33}{7}\times\frac{32}{8}\times\frac{31}{9}\times\frac{30}{10}\times\frac{29}{11}}{\frac{51}{1}\times\frac{50}{2}\times\frac{49}{3}\times\frac{48}{4}\times\frac{47}{5}\times\frac{46}{6}\times\frac{45}{7}\times\frac{44}{8}\times\frac{43}{9}\times\frac{42}{10}\times\frac{41}{11}\times\frac{40}{12}}=$$

$\frac{20112672528}{158753389900}$ the probability of one precisely.

$$\frac{\frac{12}{1}\times\frac{11}{2}\times\frac{39}{1}\times\frac{38}{2}\times\frac{37}{3}\times\frac{36}{4}\times\frac{35}{5}\times\frac{34}{6}\times\frac{33}{7}\times\frac{32}{8}\times\frac{31}{9}\times\frac{30}{10}}{\frac{51}{1}\times\frac{50}{2}\times\frac{49}{3}\times\frac{48}{4}\times\frac{47}{5}\times\frac{46}{6}\times\frac{45}{7}\times\frac{44}{8}\times\frac{43}{9}\times\frac{42}{10}\times\frac{41}{11}\times\frac{40}{12}} =$$

$\frac{41959196136}{158753389900}$ the probability of two precisely. Hence $\frac{3910797436+20112672528+41959196136}{158753389900}$ $= \frac{659826661}{1587533899}$ the probability that the dealer does not hold three trumps or more besides the trump turned up. Therefore $1 - \frac{659826661}{1587533899} = \frac{927707238}{1587533899}$ the probability that he does. Hence the odds are 927707238 to 659826661, or it is 7 to 5 nearly that the dealer has three trumps or more besides the trump turned up.

(174) *By* Mr. R. Smith, *Manchester.*

At the game of Whist, what are the odds that neither of the two sides has the four honours?

Answered by the Proposer.

In the first place, it is plain, since each player receives $\frac{1}{4}$ of the pack, or 13 cards, that for the partners, one of whom deals, to have the 4 honours, the dealers must turn one up, and they must receive 3 in 25 cards; or else, the dealer must not turn one up, and they must receive 4 in 25 cards.

It is clear before the head is cut there are 16 honours; and after, only 4; therefore the probability of the dealer turning up one is $\frac{16}{52}$; and admitting this event happens, then there remain 51 cards amongst which are 3 honours. The probability of receiving the three honors in 25 of these 51 cards is (from Ex. 4. prob. 6, Simpson's Laws of Chance,) $\frac{25}{51}\times\frac{24}{50}\times\frac{23}{49} = \frac{92}{833}$.

* * *

Therefore the probability of turning one up, and of having the other three is $\frac{16}{52} \times \frac{92}{833} = \frac{368}{10829}$.

Again, the probability of not turning up an honor is $\frac{36}{52}$; and admitting that one is not turned up, then there remain 51 cards, amongst which are 4 honors. The probability of receiving 4 honors in 25 of these 51 cards is $\frac{25}{51} \times \frac{24}{50} \times \frac{23}{49} \times \frac{22}{48} = \frac{253}{4998}$; therefore the probability of not turning an honour up, and of having 4 is $\frac{36}{52} \times \frac{253}{4998} = \frac{2277}{64974}$.

Hence $\frac{368}{10829} + \frac{2277}{64974} = \frac{115}{1666}$ the probability of the dealer and his partner having 4 honors.

In the second place, that the partners, who do not deal may have 4 honours, the dealer must not turn one up, (the probability of which is $\frac{36}{52}$) and they must receive 4 in 26 cards; and the probability of receiving 4 honors in 26 of 51 cards is $\frac{26}{51} \times \frac{25}{50} \times \frac{24}{49} \times \frac{23}{48} = \frac{299}{4998}$; therefore the probability of the partners who do not deal having the 4 honours is $\frac{36}{52} \times \frac{299}{4998} = \frac{69}{1666}$.

Hence $\frac{115}{1666} + \frac{69}{1666} = \frac{92}{833}$ is the probability that one of the two sides has 4 honours.

Again, $1 - \frac{92}{833} = \frac{741}{833}$ the probability that neither side has 4 honours.

Therefore the odds are 741 to 92, or it is $8\frac{5}{92}$ to 1 that neither side has 4 honors,

Corollary. It appears from the above that before the trump is turned up the odds are 115 to 69, or it is 8 to 5 nearly in favor of the dealer and his partner. Also, that if an honor be turned up, the dealer and his partner will have the same probability of having 4 honors as both sides had before the trump was turned up; and therefore the odds are $8\frac{5}{92}$ to 1 against them in this case.

An answer was received from Mr. Whitehead.

(175) *By* Mr. W. Shillito, *Manchester*.

Find the length of the curve whose equation is $(x^2 + y^2)^2 = a^2 \times (x^2 - y^2)$. See the solutions of question 366, Math. Companion for 1817.

Answered by Mr. J. Baines, *Nottingham.*

Put $x^2+y^2=z^2$, then $a^2.(x^2-y^2)=z^4$, or $x^2-y^2=\frac{z^4}{a^2}$; and by addition, subtraction and evolution $x=\frac{z}{2a}\sqrt{a^2+z^2}$, and $y=\frac{z}{2a}\sqrt{a^2-z^2}$; hence $x'=z'.\frac{\frac{1}{2}+\frac{z^2}{a^2}}{\sqrt{\frac{1}{2}+\frac{z^2}{2a^2}}}$, and $y'=z'.\frac{\frac{1}{2}-\frac{z^2}{a^2}}{\sqrt{\frac{1}{2}+\frac{z^2}{2a^2}}}$ therefore $\sqrt{x'^2+y'^2}$ $=z'\sqrt{\frac{1}{1-\frac{z^4}{a^4}}}=\frac{z'}{a^2\sqrt{a^4-z^4}}=$ the fluxion of the rectification. Put $a^2=b$, and $z^2=v$; then $\frac{z'}{a^2\sqrt{a^4-z^4}}$ $=\frac{v'}{2bv^{\frac{1}{2}}\sqrt{b^2-v^2}}$, whose fluent is found at Art. 355, Ex. 2, Davis's edition of Simpson's Fluxions.

Also by the Proposer.

Assume $x^2+y^2=a^2z^2$, then we shall have x^2-y^2 $=a^2z^4$ hence $x=\frac{a}{\sqrt{2}}\times\sqrt{z^2+z^4}$ $x'=\frac{a}{\sqrt{2}}\times\frac{z'+2z^2z'}{\sqrt{1+z^2}}$

$$y=\frac{a}{\sqrt{2}}\times\sqrt{z^2-z^4},\ y'=\frac{a}{\sqrt{2}}\times\frac{z'-2z^2z'}{\sqrt{1-z^2}}.$$

Whence $\sqrt{x'^2+y'^2}=\frac{az'}{\sqrt{1-z^4}}$, being the fluxion of the curve of the Lemniscata whose semi-axis $=a$, and the length of any chord drawn from its centre $=az$, the fluent of which is found in Art. 980 of Maclaurin's Fluxions, and in other places.

Mr. Garside also sent an ingenious solution.

(176) *By* Mr. W. Shillito, *Manchester.*

Rectify the curve whose equation is $(x^2+y^2)^2 = d^2 xy$: determine its greatest ordinate, and shew that this curve and that in the preceding question (175) are identical. See Dr. Puff's solution of question 363, Math. Companion, 1817.

Answered by the Proposer.

Let $4a^2 = d^2$, and $x^2+y^2 = z^2$, then $z^4 = 4a^2xy$, or $2xy = \frac{z^4}{2a^2}$. Hence

$$x+y = \sqrt{z^2 + \frac{z^4}{2a^2}},\ \dot{x}+\dot{y} = \frac{a^2\dot{z} + z^2\dot{z}}{a\sqrt{a^2+\frac{z^2}{2}}}$$

$$x-y = \sqrt{z^2 - \frac{z^4}{2a^2}},\ \dot{x}-\dot{y} = \frac{a^2\dot{z} - z^2\dot{z}}{a\sqrt{a^2-\frac{z^2}{2}}}.$$

Whence $\sqrt{\dot{x}^2+\dot{y}^2} = \sqrt{(\dot{x}+\dot{y})^2 + (\dot{x}-\dot{y})^2 \div 2} =$ $\frac{a^2\dot{z}}{\sqrt{a^4-\frac{z^4}{4}}}$ = (putting $\frac{z^2}{2} = a^2v^2$, or $z = av\surd 2$) to $a\surd 2 \times \frac{\dot{v}}{\sqrt{1-v^4}}$; which being the same form as that in the preceding question, shews that the two curves are identical.

To find the greatest ordinate. Assume $x = v^2$ and $y = z^2$ then the equation of the curve becomes $(v^4 + z^4)^2 = d^2v^2z^2$, or $v^4 + z^4 = dvz$, or $v^4 + z^4 - dvz = 0$ (A). Take the fluxion of this equation, z being constant, and $4v^3v' - dv'z = 0 \therefore z = \frac{4v^3}{d}$: write this value of z in equation A, and we have $v^4 + \frac{256v^{12}}{d^4} - 4v^4 = 0 \therefore 256v^8 = 3d^4$, or $256x^4 = 3d^4 \therefore x = \frac{d}{4}\sqrt[4]{3}$: therefore $y = z^2 = \frac{16v^6}{d^2} = \frac{16x^3}{d^2} = \frac{d}{4}\sqrt[4]{27}$, the greatest ordinate required.

Mr. Baines also sent an answer.

(177) *By* Mr. S. TREEBY, *Schoolmaster, Plymouth.*

Given the base AB, and the vertical angle C, of a plane triangle, to determine geometrically the nature of the curve which a variable point P in the side BC describes when AC has to BP a given ratio.

Answered by Mr. J. H. SWALE.

Describe through A, B, the extremities of the given base AB, a circle to contain the given vertical ∠; this circle, the centre of which let be L, will be the locus of C. While then the lines AC, BC, revolve about the points A, B, and have their point of concourse C in the periphery of the given circle ACB, we have to determine the LOCUS of a point P in BC, so that AC : BP shall be a given constant ratio. Take CQ = BP, and draw AQ; then CA : CQ = a given ratio, and ∠ACQ = a given ∠, or the △ ACQ given in species, and of course CQA, BQA, are given ∠s; that is, the locus of Q is a given circle, passing through A, B; let this circle be described, draw its diameter BM, and join MQ, ML; make LK = LM, join KP, and draw LR to R, the point of bisection of PQ.

The circle BQA is given in magnitude and position, and BM is a diameter, passing through a given point B, or M a given point; also L, being the centre of the given circle ACB, is a given point, or ML given in length and position; and as LK = LM, LK is given in length; it is also given in position, MLK being a continued line; then K is a given point. Moreover, since CQ = BP, we have CP = BQ, and as PR = RQ, it follows that CR = RB. The line LR is therefore at right $\angle$s to BC; that is, MQ, LR, are parallel to each other; also, because of equals, ML : LK = QR : RP, or KP par. to MQ or LR; of course $\angle$ BPK is a right $\angle$. Whence, B, K, are given points, about one of which, as B, revolves the right line BP, while through the other point K, is drawn the right line KP, at right $\angle$s to BP, and we are required to determine the curve which is the locus of P. Draw BK, upon which, as a diameter, describe the circle BPK, for the curve to be determined.

Again by Mr. W. Stringer.

On the given base AB describe the segment ACB of a circle to contain the given vertical angle, and to the point B draw the tangent BF so that AB may have to BF the given ratio of AC to BP; on BF describe a segment BPF of a circle similar to the segment ACB, which shall be the locus of P required. For the angles ACB, BPF are equal by construction, and by Eu. 32, 3, the angles FBP, BAC are likewise equal, and the triangles ACB, FPB similar; therefore AB : BF :: AC : BP, the given ratio.

Ingenious solutions to this question were received from Messrs. Garside and Whitehead.

(178) *By* Mr. J. H. Swale, *Liverpool.*

Given the vertical angle, line bisecting the base, and the sum or difference of the perpendicular, and a line to which the base has a given ratio, to determine the triangle.

Answered by Mr. J. H. SWALE.

Suppose BLA the △ to be determined, and LE the line bisecting the base BA, make EP perpendicular to BA, and BA : EP = the given ratio; draw PA, PB, to meet which in F, G, and PE in D, draw, through L, a parallel to AB.

Since PE is the line to which the base has the given ratio, and ED = the perpendicular from L upon AB, it follows that PD and the isosceles △ PFG, are given. It remains to interpose, between PD, DF, the given line EL so that drawing AB through E, par. to FG, and joining LA, LB, the ∠ BLA shall be of a given magnitude. Draw FC par. to AL, meeting PL in C, join GC, DC, and draw CH perpendicular to PD: It need not be proved, that GC, DC, will be par. to BL, EL; or GCF = a given ∠, and PH : DC = PD : EL a given ratio. Now, as FG is given, and ∠ FCG of a given mag. the circle passing through C will be given in magnitude and position, and P, D, given points in its diameter; our prob. is, to draw DC so, that demitting the perpendicular CH upon PD, the ratio DC : PH may be given. To this prob. a variety of solutions may be given: it is easily reduced, like quest. 10, to a case of the *sectio rationis*: we shall pursue it differently, as follows: In DF take DK = EL, let PK meet HC at M; then PM is given in pos., HM = DC; and we have to draw HCM and DC, making HM = DC. We may now give the following easy

COMP. Draw QI perpendicular to PT, Q being the centre of the circle on FG and T its intersection with PM, meeting DF in R, the circle to centre R and radius RT, determines N, in DR; the tangent NM determines M, in PF, and of course the whole. Draw CD, MR, TR; we have to prove that HM = DC.

By the Comp. RN = RT, or $RN^2 = RT^2$, and $MR^2 = MN^2 + NR^2 = MN^2 + RT^2 = MI^2 + IR^2$; therefore $MI^2 - MN^2 = RT^2 - RI^2 = IT^2$, and MI^2

— $IT^2 = MN^2 = MH^2 - HC^2 = HD^2$ for $MN = HD$; that is $MH^2 = DH^2 + HC^2 = DC^2$ per 47. E. 1. Consequently $MH = DC$, and the rest obvious.

When the difference is given P will fall on the contrary side of AB; and P falls between E, D, when EP is less than ED. Draw the tangent MV to the circle FCG, then $MN = MV$. Also this prob. is a particular case of the following: To determine a point in a right line given in pos. so, that drawing a tangent to a given circle, and also a right line making a given $\angle$ with another right line given in pos. the tangent and line, so drawn, shall have a given ratio.

Mr. Garside also answered this question.

(179) *By* Mr. S. Johnson.

Given the radius of the inscribed circle, the rectangle of the segments of the base made by the contact of the said circle, and the sum of the radii of the two circles inscribed in each of the triangles made by a line drawn from the verticle angle to the middle of the base, to construct the plane triangle.

By Mr. J. H. Swale,* *Brunswick-place Academy, Liverpool.*

Suppose the $\triangle$ ACB constituted with the proposed data: that is, the radius QD of the inscribed circle, the rectangle of the segments AD, DB, by the point of contact D, and the sum of the radii PN, OM, of the circles PNM′, OMN′, inscribed in the $\triangle$s BCF, ACF, formed by the bisecting line CF and the sides of the triangle.

Draw FE perpendicular to AB, meeting CQ at E, make $ER = EQ$, and draw RG perpendicular to AB, meeting QH, CK, drawn parallel to BA at H, K; and join EB: the intersection of AB, CE, being at L′.

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* Mr. Swale is on the point of publishing an Octavo Volume entitled "Geometrical Amusements."

It is well known, that $AG = BD$, $GB = DA$, and the points A, H, Q, B, R, in the periphery of the circle to centre E and radius EQ: or $AD.DB = AG.GB = RG.GH = RG . DQ$ = a given rectangle, and DQ a given line, or RG given; that is, EF is given; for, because of the equals EQ, ER, we have $2EF, = RG - QD$.

Also, $EB : QC = EQ : QC = EF : QD$, and $EQ : QL' = EF + DQ : DQ$, are given ratios; or $QL' : QC = HG : HK = QD : HK$ a given ratio; of course HK, GK, RK are given lines.

Draw OP, bisect it in L, draw LI perpendicular to AB, join FL, produce DQ to FL, FC, at S, T; and draw ON′, PM′, LI′, SV, perpendicular to FC, producing CF to KR at Q′.

Per a known property of △s, $AF + FC - AC = 2MF$, and $BF + FC - BC = AF + FC - BC = 2NF = 2ND + 2DF$; or $AC - CB = 2ND + 2DF - 2MF = 2FD$; that is, $2ND = 2MF$, or $ND = MF$, and OL being equal LP, it follows that $MI = IN$, $FI = ID$, and therefore $DS = 2IL = PN + OM$ = a given line. We also readily shew that $PN - OM = SV$; and that Q′, the point in RK through which CF passes, is a given point. For since $OL = LP$, we shall have, because of parallels, $PM' - ON' = PN - OM = 2LI'$; also, $FL = LS$, or $FS = 2FL$, and consequently, $SV = 2LI'$ or $PN - OM$; moreover, $FD = FG$, and therefore per parallels, $GQ' = DT$ a given line; for EF, DQ, are given lines, and $CE : CQ$ a given ratio; that is, Q′ is a given point. Demit upon CA, CB, the perpendiculars RW, RU. It need not be proved, that $RW = RU = RG$, and $2CW = 2CU = AC + CB + AB$; and the △s BFC, AFC, being equal, we shall have $PN : OM = AF + FC + AC : BF + FC + BC$; Componendo et dividendo, $PN + OM : PN - OM = 2AF + 2FC + AC + BC : AC - BC = AC + CB + AB + 2FC : AC - BC = 2CW + 2CF : 2FD = CW + CF : FD = 2IL : SV = DS : SV$, Whence, $CW + CF : DS = FD : SV = FT : TS$, per the equiangular △s FDT, SVT.

Hence, $CW + CF : FT = DS : ST =$ a given ratio; and as $CF : FT$ and $CF : CQ'$, are also given ratios; it follows that, $CW + CF : CF$, $CW : CF$, and $CW : CQ'$, are given ratios; and our problem now assumes the following pleasing and simple form; the circle WGU, the right line KC, and the point Q', are given in position, to determine the point C, in KC, so, that drawing the tangent CW to the given circle, and the right line CQ' to the given point; the ratio $CW : CQ'$ may be given.

In the present particular case of the general prob., where Q' is posited in RK, the perpendicular from the centre R upon CK, the line given in position, the determination will be extremely easy.

Take, in KR, the line $KP' = KV'$, the tangent drawn from K, to the given circle, and draw $P'C$. Now, whatever position the point C has in KC, it is known, that $CP' = CW$*; it only remains then, to draw, from the given points P' Q', the lines $P'C$, $Q'C$, making $P'C : Q'C$ a given ratio. Hence, as $P'KC$ is a right ∠, we have the ratio and the diff. of the squares of the lines $P'C$, $Q'C$, to determine them.

It would be trifling to add the comp. of this very easy prob. When the diff. or ratio of PN, OM, is one of the data, methods of determination might be pointed out; also, several curious properties of the fig. might be added, had we not already exceeded the limits usually assigned to a solution.

By the Proposer.

Lemma. If ACB be any plane triangle, CK the line bisecting the base AB, G the contact of the inscribed circle with AB, R and Q the points of contact of the circles inscribed in the triangles ACK, and BCK with the base, I say RK is equal to GQ.

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* See Question XII. Ladies Diary, 1816.

Demon. From the centres F and P demit on AB, CK the perpendicular radii FR, Fx, PQ, Pz, and bisect CK in V. By the property of the inscribed circles AC — AK $=$ 2Vx, and BC — BK (AK) $=$ 2Vz; therefore AC — BC $= 2xz =$ 2KG, or $xz =$ KG. Again by the property of tangents to the circle, KR $=$ Kx, also KQ $=$ Kz, and since $xz =$ KG, therefore KR $=$ GQ.

Now let ACB represent the required triangle, Cy the line bisecting the base, DQ the radius of the inscribed circle perpendicular to the base AB, also let GE, and FK be the radii of the circles inscribed in the triangles ACy and BCy perp. to AB; join AD, BD and draw ST parallel to DQ and equal to $\frac{1}{2}$ (GE + FK) also par. to AB draw FH, SP, GL cutting DQ in a, R, u, respectively, and the other lines as in the figure. Then by similar triangles

$$aF(QK) : aD(DR - VS) :: QB : DQ$$
$$Gu(QE) : uD(DR + VS) :: AQ : DQ$$
$$\underline{GE(TS - VS : Ey(QK) :: Ky(QE) : FK(TS + VS)}$$

Wherefore $TS^2 - VS^2 : DR^2 - VS^2 :: AQ.BQ : DQ^2$; but AQ . BQ, DQ^2, TS^2, and DR^2 are given magnitudes; hence VS becomes known, and consequently the radii GE and FK are determined. Again by parallel lines and the Lemma $AE = BK + Qy : EQ = Ky :: GE : Du$, therefore dividendo et invertendo $GE - Du : Du :: BK - QK : Ky$, and similarly we shew that $Da : FK - Da :: QK : BK - QK$ therefore by multiplication $Da . (GE - Du) : Du (FK - Da) :: QK : Ky$, a given ratio; therefore the rectangle QK . Ky or its equal the rect. FK . GE has to Ky^2 a given ratio, and since FK . GE is given, Ky^2, or Ky is given. *Ergo solutum.*

Mr. Johnson also gave an ingenious algebraical solution.

(180) *By* VERUS, *from London.*

Find the sum of x terms of the series $3.2^3 + 8.3^3 + 15.4^3 + 24.5^3 +$ &c. by the method of increments.

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By the Proposer.

The general term or increment of the series is $(x^2+2x)\times(x+1)^3=(x+1)^5-(x+1)^3=x^5+5x^4+9x^3+7x^2+2x$. Hence by taking the sums of of the integrals

$$\begin{array}{rl}
\S' x^5 = & \frac{x^6}{6}-\frac{x^5}{2}+\frac{5x^4}{12} \quad * \quad -\frac{x^2}{12} \quad * \\
5\S' x^4 = & \ldots\ldots + x^5-\frac{5x^4}{2}+\frac{5x^3}{3} \quad * \quad -\frac{x}{6} \\
9\S' x^3 = & \ldots\ldots\ldots\ldots + \frac{9}{4}x^4-\frac{9}{2}x^3+\frac{9}{4}x^2 \quad * \\
7\S' x^2 = & \ldots\ldots\ldots\ldots\ldots\ldots + \frac{7}{3}x^3-\frac{7}{2}x^2+\frac{7}{6}x \\
2\S' x = & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + x^2-x \\
\hline
& \frac{x^6}{6}+\frac{x^5}{2}+\frac{x^4}{6}-\frac{x^3}{2}-\frac{x^2}{3} = \text{sum of } x-1
\end{array}$$

terms as required.

By Mr. JAMES NORTH.

Assume $[(x+1)^2-1]\times(x+1)^3$ or $(x^5+5x^4+9x^3+7x^2+2x=Ax^6+Bx^5+Cx^4+Dx^3+Ex^2+Fx$.

Then by the method of increments we have

$$\begin{array}{rl}
\Delta' Ax^6 = & 6Ax^5+15Ax^4+20Ax^3+15Ax^2+6Ax+A. \\
\Delta' Bx^5 = & \ldots\ldots + 5Bx^4+10Bx^3+10Bx^2+5Bx+B. \\
\Delta' Cx^4 = & \ldots\ldots\ldots\ldots + 4Cx^3+6Cx^2+4Cx+C. \\
\Delta' Dx^3 = & \ldots\ldots\ldots\ldots\ldots\ldots + 3Dx^2+3Dx+D. \\
\Delta' Ex^2 = & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2Ex+E. \\
\Delta' Fx = & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots +F. \\
\hline
= & x^5+5x^4+9x^3+7x^2+2x.
\end{array}$$

Whence equating the co-efficients of the homologous terms

$6A=1$, or $A=\frac{1}{6}$.

$15A+5B=5$, or $B=\frac{1}{2}$.

$20A+10B+4C=9$, or $C=\frac{1}{6}$,

$15A+10B+6C+3D=7$, or $D=-\frac{1}{2}$,

$6A+5B+4C+3D+2E=2$, or $E=-\frac{1}{3}$.

$A+B+C+D+E+F=0$, or $F=0$. Thus

upon the whole the sum of $x - 1$ terms of the proposed series is $= \frac{1}{6}x^6 + \frac{1}{2}x^5 + \frac{1}{6}x^4 - \frac{1}{2}x^3 - \frac{1}{3}x^2$.

Messrs. Godward and Gillott also sent solutions.

(181) *By* J. C. *Bath.*

If a rectangle be inscribed in a plane triangle so that its diagonals shall be the least possible: the time of descent down either diagonal shall be equal to the time of descent down the perpendicular height of the triangle. Required a demonstration.

By Mr. J. H. SWALE, *Liverpool.*

To inscribe a rectangle in a plane △ ABC, the diagonal of which shall be the least possible. Draw BL perpendicular to the base AC, make LP = CA, draw BP, and LQ perpendicular to BP: then QDG, drawn par. to PC, determines the angular points D, G, of the rectangle. Draw GF perp. to AC, and join FD: DG cutting BL in K. By the comp. LP = CA, or PA = LC, and therefore QD = KG = LF; that is, QDFL is a parallelogram, having QL = DF: and as LQ is the least possible, being a perpendicular upon BP, it follows that FD is also the least possible. This comp. is more elegant than that at p. 204, Vol. II, Leeds Corres.

When the diagonal, instead of being the least possible, is to be equal to a given line. Having drawn BL, BP, as above, apply LR, to BP, equal to the given line: then a line from R, par. to AC, will determine the angular point as before. The truth of the theorem follows so obviously from the comp. for the minimum value of DF, that a Demonstration seems superfluous. For BQL being a right ∠, the times of descent down QL and BL, are evidently equal; and as QL and DF are equal and parallel, the prop. is manifest.

We deduce the following simple theorem, which we add for the sake of the young Geometrician.

If any rectangle whatever, as DGFH, be inscribed in any plane △ ABC, the perpendicular BL demitted upon the base AC; BP drawn, making LP = CA, and GD produced to BP at Q: then LQ will be parallel to FD, the diagonal of the rectangle.

Ingenious solutions were also given by Messrs. Baines, Garside, Lightbourn, and Stringer.

(182) *By* Amicus.

A pin three inches long is fixed in the centre perpendicular to a horizontal circular table on which a candle is placed burning constantly at the rate of 3 inches per hour. Now supposing the candle to revolve vertically and uniformly 15 times round the centre of the table at the distance of 15 inches from it in 3 hours; it is required to determine the several lengths of the curve described by the extremity of the pin's shadow on the table at the end of the 1st, 2nd, 3rd, 4th &c. revolution of the candle, the apex of the flame being 12 inches above the table at the commencement of the motion.

Answered by Mr. J. Whitehead, *Huddersfield*, and Mr. Samuel Johnson, *Apperley-Bridge*.

In the figure let AB represent the plane of the table, FE the pin, AF the distance of the candle from the centre, AC its height at any given time t, and let x denote the arc of a circle to rad. 1 and circumference $a = 6.2832$, described by a point in the time t, and making a revolution in the same time as the candle. Then since the candle makes each revolution in 12 minutes therefore $a : 12' :: x : \frac{12x}{a} = t$: and because it burns at the rate of 3 inches per hour $\therefore 60' : 3 :: \frac{12x}{a} : \frac{3x}{5a} =$ the decrease of the height in the time t: hence $9 - \frac{3x}{5a} = \frac{3(15a - x)}{5a} =$ CD, and per similar triangles CD : DE (15) :: FE (3)

: FB, or FB $= \dfrac{75a}{15a - x} = y$. But the fluxion of the spiral's length is $\dot{z} = \sqrt{y^2\dot{x}^2 + \dot{y}^2}$ (Simpson's Flux. Davis' Ed. art. 136) $= \left(\dfrac{5625a^2\dot{x}^2}{(15a-x)^2} + \dfrac{5625a^2\dot{x}^2}{(15a-x)^4}\right)^{\frac{1}{2}} = \dfrac{75a\dot{x}}{(15a - x)^2}\sqrt{(15a - x)^2 + 1} = -\dfrac{75a\dot{v}}{v^2}\sqrt{v^2 + 1}$, (puttting $v = 15a - x$): the correct fluent of which is $z = 75a \times \left\{\dfrac{\sqrt{v^2 + 1}}{v} - \dfrac{\sqrt{225a^2 + 1}}{15a} + \text{hyp. log.}\ \dfrac{15a + \sqrt{225a^2 + 1}}{v + \sqrt{v^2 + 1}}\right\}$. Whence expounding v by $14a$, $13a$, $12a$, &c. respectively, we obtain the corresponding values of z at the end of the 1st, 2nd, 3rd, &c. revolution of the candle as follows, viz. when

$x = a$, $z = 32.5142$, $x = 5a$, $z = 191.088$, $x = 9a$, $z = 431.859$, $x = 13a$, $z = 950.2$

$x = 2a$, $z = 67.4394$, $x = 6a$, $z = 240.745$, $x = 10a$, $z = 517.815$, $x = 14a$, $z = 1279.1$

$x = 3a$, $z = 105.1618$, $x = 7a$, $z = 296.259$, $x = 11a$, $z = 623.037$, $x = 15a$, $z =$ infinity.

$x = 4a$, $z = 146.1689$, $x = 8a$, $z = 359.194$, $x = 12a$, $z = 758.749$.

AMICUS, the proposer, solves the problem in exactly the same manner; and further adds "that as the computation derived from the preceding formula is very elaborate, a simple and nearly approximate solution may be obtained by rejecting the units under the *vincula*: in consequence of which the general expression for the length of the curve is $z = 75a \times \text{hyp. log.}\ \dfrac{15a}{v}$: whence writing for v the several values $14a$, $13a$, $12a$, &c. we shall have the corresponding values of z at the end of the 1st, 2nd, 3rd, &c. revolution as below, viz. when

$x = a$, $z = 75a \times \text{h. l.}\,\frac{15}{14} = 32.1$, $x = 9a$, $z = 75a \times \text{h. l.}\,\frac{15}{6} = 431.8$.

$x = 2a$, $z = 75a \times \text{h. l.}\,\frac{15}{13} = 67.3$, $x = 10a$, $z = 75a \times \text{h. l.}\,\frac{15}{5} = 517.7$.

$x = 3a$, $z = 75a \times \text{h. l.}\,\frac{15}{12} = 105.1$, $x = 11a$, $z = 75a \times \text{h. l.}\,\frac{15}{4} = 622.8$.

$x = 4a$, $z = 75a \times \text{h. l.}\,\frac{15}{11} = 146.1$, $x = 12a$, $z = 75a \times \text{h. l.}\,\frac{15}{3} = 758.4$.

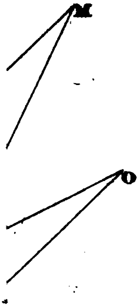
$x = 5a$, $z = 75a \times \text{h. l.}\,\frac{15}{10} = 191.1$, $x = 13a$, $z = 75a \times \text{h. l.}\,\frac{15}{2} = 949.5$.

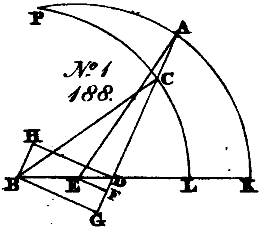
$x = 6a$, $z = 75a \times \text{h. l.}\,\frac{15}{9} = 240.7$, $x = 14a$, $z = 75a \times \text{h. l.}\,\frac{15}{1} = 1276$.

$x = 7a$, $z = 75a \times \text{h. l.}\,\frac{15}{8} = 296.2$, $x = 15a$, $z = 75a \times \text{h. l.}\,\frac{15}{0} = \text{infinity}$.

$x = 8a$, $z = 75a \times \text{h. l.}\,\frac{15}{7} = 359.1$.

III. Jan. 1820.

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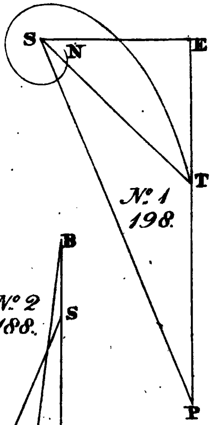
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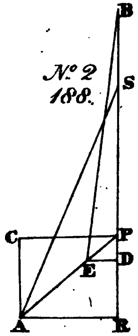
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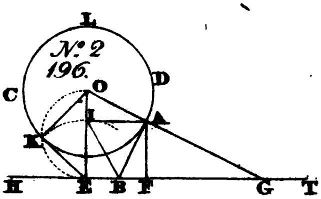
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UNIV
OF

THE

LEEDS CORRESPONDENT,

A

Literary, Mathematical, and Philosophical Miscellany.

> Tarda fluunt ingrataque tempora, quæ spem
> Consiliumque morantur agendi gnaviter id, quod
> Æque pauperibus prodest, locupletibus æque,
> Æque neglectum pueris senibusque nocebit.
>
> HORAT.

No. II. JULY, 1820. Vol. III.

Translations are respectfully requested of those, among our junior correspondents, who have not attained to sixteen years of age. The extracts in each language are divided into two or three portions; that any youth, not wishing to translate the whole, may have an opportunity of applying his powers to a single portion.

JUVENILE DEPARTMENT.

Des Pauvres.

QUELS sont les services que nous devons rendre aux pauvres? en quoi sommes-nous tenus de les assister? L'admirable St. Augustin nous donne sur ce sujet là une instruction particulière. Le service que vous devez aux nécessiteux, nous dit-il, est de porter avec eux une partie du fardeau qui les accable. L'apôtre St. Paul ordonne aux fidèles de porter les fardeaux les uns des autres. Les pauvres ont leur fardeau, et les riches aussi ont le leur. Les pauvres ont leur fardeau, qui ne le sait pas? Quand nous les voyons suer et gémir, pouvons-nous ne pas reconnoître que tant de miserès pressantes sont un fardeau très-pesant dont leurs épaules sont accablées? Mais encore que les riches marchent à leur aise, et semblent n'avoir rien qui leur pèse, sachez qu'ils ont aussi leur fardeau. Et quel est ce fardeau des riches? Hourriez-vous le croire? Ce sont leurs propres richesses. Quel est le fardeau des pauvres? C'est le besoin. Quel est le fardeau des riches? C'est l'abondance. Le fardeau des pauvres, dit St. Augustin, c'est de n'avoir pas ce qu'il faut; et le fardeau des riches, c'est d'avoir plus qu'il ne faut. Quoi donc? est-ce un fardeau incommode que d'avoir trop de biens? Ah! qu'il y a de mondains qui désirent un tel fardeau dans le secret de leur cœurs! Mais qu'ils arrêtent ces désirs inconsidérés. Si les injustes préjugés du siècle les empêchent de concevoir en ce monde combien l'abondance pèse, quand ils iront dans ce pays, où il nuira d'être trop riches, quand ils comparoîtront à ce tribunal où il faudra rendre compte non-seulement des talens dispensés, mais encore des talens enfouis, et répondre à ce juge inexorable non-seulement de la dépense, mais encore de l'épargne et du ménage; alors ils connoîtront que les richesses sont un grand poids et ils se repentiront de ne s'en être pas déchargés.

Le Besoin, Père des Arts.

Tel est l'arrêt fatal du maître du tonnerre:
Lui-même il força l'homme à cultiver la terre;
Et n'accordant ses fruits qu'à nos soins vigilans,
Voulut que l'indigence éveillât les talens.
Avant lui, point d'enclos, de bornes, de partage;
La terre étoit de tous le commun héritage,
Et sans qu'on l'arrachât, prodigue de son bien,
La terre donnoit plus à qui n'exigeoit rien.
C'est lui qui, proscrivant une oisive opulence,
Partout de son empire exila l'indolence:
Il endurcit la terre, il souleva les mers,
Nous déroba le feu, troubla la paix des airs,
Empoisonna la dent des vipères livides,
Contre l'agneau craintif arma les loups avides,

Dépouilla de leur miel les riches arbrisseaux,
Et du vin dans les champs fit tarir les ruisseaux.
Enfin, l'art, à pas lents, vint adoucir nos peines;
Le caillou rend le feu recelé dans ses veines;
La terre obéissante et les flots étonnés,
Par la rame et le soc déjà sont sillonnés;
Déjà le nocher compte et nomme les étoiles;
Des chiens lancent un cerf, le chasseur tend ses toiles;
La glu trompe l'oiseau; le crédule poisson
Tombe dans les filets, ou pend à l'hameçon:
Bientôt le fer rougit dans la fournaise ardente;
J'entends crier la dent de la lime mordante;
L'acier coupe le bois que déchiroient les coins,
Tout céde aux longs travaux, et surtout aux besoins.

A PICTURE OF FALSE AND TRUE VALOUR.

A Translation from the French.

GENTLEMEN,

Do not understand by the word valour, a rash and precipitate resolution, which seeks danger for the sake of danger; exposes itself without profit, and has no loftier aim than reputation and the vain applause of men

I speak of a wise and governed resolution, which is animated at the sight of its enemies; which, even in danger, provides for every thing, and avails itself of every advantage; but which compares itself with its powers; undertakes things that are difficult; attempts none that are impossible; and abandons nothing to the hazard of that which may be conducted by virtue: Adequate, in short, to every enterprise, when deliberation is useless; and ready, in the fulfilment of its duties, either to die in victory, or survive its defeat.

Rochdale. HENRY LITTLEWOOD.

AN ARMY.

A Translation from the French.

What is an army?—It is a body animated by an infinitude of various passions, which a skilful man causes to move for the defence of their country. It is a troop of armed men, who blindly follow the directions of a chief, of whose intentions they are ignorant. It is a multitude of minds for the most part vile and mercenary, who, forgetting their own reputation, labour for that of kings and conquerors. It is a confused assemblage of libertines who must be made subject to obedience; of cowards who must be led to battle; of rash persons whom it is necessary to restrain; and of unstable ones whom it is requisite to accustom unto firmness.

Rochdale, HENRY LITTLEWOOD.

THE PATERNAL AND MATERNAL INSTINCT OF BIRDS.

A Translation from the French.

O THOU who darest to deny
The God who form'd the earth and sky,
Come, and elucidate to me
This artful piece of masonry;
In which such order is display'd,
And which, with equal taste array'd,
The swift-wing'd swallow and her mate
Have join'd their bills to elevate.
How, this strong building to erect,
Has each laborious architect
Together heap'd the hollow mound,
And form'd the nest so neatly round?
Why are they able to foresee
Their tender young's nativity?
On many a verdant branch put forth,
Suspended cradles wait their birth,

Of softer silken down compos'd
Than that on which man e'er repos'd.
Afar the anxious father flies
Of provender for fresh supplies.
The air an easy passage yields,
Nor long confines him from those fields
From which he bears, with wing elate,
The gleanings to his tender mate.
She equally divides the food
Among her little chirping brood.
With dauntless breast and watchful eye,
They guard against each enemy,
That would by rude and fierce inroad,
Disturb the peace of their abode.
These young which with maternal care
She watches, shall (if God may spare,)
The same parental love return
To children of her children born.
And when, the nuptial torch to fire,
Returning zephyrs shall conspire,
United by the tend'rest ties
Their families shall fill the skies;
Where soon the many infant choirs
Their fathers, and their fathers' sires,
Shall no more know; but all shall be
Mingled in one great family.
Nor, when cold winter ends our year,
Will those who seek a warmer sphere,
Suffer their idle troop to stay
Within the rig'rous season's sway.
The chiefs convoke a council wise
To fix the day when all shall rise,
To seek some distant warmer shore:
That day arrives, away they soar.
He who one summer has but known,
Casting his little eyes adown,
Asks of the bird which gave him birth,
"O when shall spring revive the earth?
"That spring which shall our longing broods
"Recall to their paternal woods?"

London. JOHN M. HARE.

TIMÆUS, SEU DE UNIVERSO.

Qua ex conjunctione cælum ita aptum est, ut sub adspectum et tactum cadat. Itaque et ob eam causam, et ab iis rebus numero quatuor mundi est corpus effectum, ea constructum proportione, qua dixi: ex quo ipse se concordi quadam amicitia, et caritate complectitur, atque ita aptè cohæret, ut dissolvi nullo modo queat, nisi ab eodem, à quo est colligatus. Earum autem quatuor rerum, quas suprà dixi, sic in omni mundo omnes partes collocatæ sunt, ut nulla pars hujusce generis excederet extrà, atque in hoc universo inessent genera illa universa. Id ob eas causas, primùm ut mundus animans, possit ex perfectis partibus esse perfectus: deinde ut unus esset, nulla parte, unde alter gigneretur, relicta: postremò ne quis morbus eum possit, aut senectus affligere, omnis enim coargumentatio corporis, vel calore, vel frigore, vel aliqua impulsione vehementi labefactatur, et frangitur, et ad morbos senectutemque compellitur.

Hanc igitur habuit rationem effector mundi, molitor que Deus, ut unum opus totum, atque perfectum ex omnibus totis, atque perfectis absolveretur, quod omni morbo seniove careret. Formam autem et maximè sibi cognatam, et decoram dedit. A quo enim animante omneis reliquos contineri vellet animanteis, hunc ea forma figuravit, qua una omnes reliquæ formæ concluduntur: et globosus est fabricatus: quod "*sphæroides*" Græci vocant: cujus omnis extremitas partibus à medio radiis attingitur: idque ita tornavit, ut nihil effici possit rotundius, nihil ut asperitatis haberet, nihil offensionis, nihil inclusum angulis, nihil amfractum, nihil eminens, nihil lacunosum: omnesque partes simillimæ omnium, quoad ejus præstabat judicio dissimilitudini similitudo. Omni autem totam figuram mundi lævitate circumdedit. Nec enim oculis egebat, quia nihil extrà, quod cerni posset, relictum erat: nec auribus, quia ne quod audiretur quidem: Neque erant anima circumfusa extrema mundi, ut respirationem requireret, nec vero desiderabat aut alimenta corporis: Aut detractionem confecti, et consumti cibi. Neque enim ulla decessio fieri poterat, neque accessio: neque verò erat unde. Ita se ipse consumtione et senio alebat sui, cum ipse per se, et à se et pateretur et faceret omnia. Sic enim ratus est ille, qui ista junxit, et condidit, ipsum se contentum esse mundum, neque egere altero. Itaque nec ei manus affinxit, quia nec capiendum quidquam erat, nec repellendum: Nec pedes, nec alia membra, quibus ingressus corpore sustineret. Motum enim dedit cælo eum, qui figuræ ejus sit aptissimus, qui unus ex septem motibus mentem, atque intelligentiam cohiberet maxime. Itaque una conversione, atque eadem ipse circum se torquetur, et vertitur. Sex autem reliquos motus ab eo separavit. Itaque eum ad omni erratione liberavit. Ad hanc igitur conversionem, quæ pedibus, et gradu non egeret, ingrediendi membri non dedit.

Hæc Deus is, qui erat, de aliquando Deo futuro cogitans, levem eum effecit, et undique æquabilem, et à medio ad summum parem, et perfectum, atque absolutum ex absolutis, atque perfectis. Animum autem ut in ejus medio collocavit, ita per totum tetendit: deinde eum circumdedit corpore, et vestivit extrinsecus: cæloque solivago, et volubili; et in orbem incitato complexus est, quod secum ipsum propter virtutem facilè esse posset, ne desideraret alterum, satis sibi ipse notus, et familiaris. Sic Deus ille æternus hunc perfectè beatum Deum procreavit. Sed animus haud ita ut modò locuti sumus, tum denique cum corpus ei effecisset, inchoavit, neque enim esset rectum, minori parêre majorem.

Sed nos multa inconsideratè, ac temerè dicimus. Deus autem et ortu et virtute antiquiorem genuit animum, eùmque, ut dominum atque imperantem, obedienti præfecit corpori: idque molitus tali quodam est modo. Ex materia, quæ individua est, et quæ semper uniusmodi, suique similis, et ex ea, quæ corporibus dividua gignitur, tertium materiæ genus ex duobus in medium

admiscuit, quod esset ejusdem naturæ: Et quod alterius: Idque interjecit inter individuum, atque id, quod dividuum esset in corpore. Et cum tria sumsisset, unam in speciem temperavit: naturamque illam quam alterius diximus vel cum eadem conjunxit, fugientem et eius copulationis alienam permiscens cum materia, cum ex tribus effecisset, unum id ipsum in ea quæ decuit, membra partitus est.

AD LICIANUM, SCRIPTORES UNDE.

VERONA docti syllabas amat vatis;
  Marone felix Mantua est;
Censetur Apona Livio suo tellus,
  Stellaque nec Flacco minus:
Apollodoro plaudit imbrifer Nilus;
  Nasone Peligni sonant.
Duosque Senecas, unicumque Lucanum
  Facunda loquitur Corduba.
Gaudent jocosæ Canio suo Gades;
  Emerita Deciano meo.
Te, Liciniane, gloriabitur nostra
  Nec me tacebit Bilbilis.

A Translation from the Latin of Cicero, being the Second Part of the Fragment entitled

When our juvenile readers meet with any passage which seems to countenance Materialism, they must recollect that Cicero was a Heathen, and not favoured with the light of the Gospel.

TIMÆUS, OR A DISCOURSE ON THE UNIVERSE.

IN relation then to what has been introduced, let this constitute the first part. Whatever be discussed, there appears to be a standard for things which are to be examined. When therefore the subject of dispute is something permanent and immutable, the language of the argument, like the subject, should be such as cannot be disproved or confuted: But when we discourse on imitative or changeable forms, we suppose that we have done well if we reach the similitude of truth. For as Eternity is paramount to what has had a beginning, so is truth to mere belief. Wherefore if when discoursing on the nature of the Gods and the origin of the world, our language should not appear altogether perspicuous and elegant, or in every thing consistent with itself, (which is what we have in view,) it will not indeed be wonderful; and you should be satisfied if probabilities be advanced. For it is proper to remember, that I who deliver my sentiments, and you who judge, are but men; so that if what is probable be advanced, you should require nothing further.

Let us then try to discover the reason which induced the Maker of these things to give them birth and to construct a new fabric. This was certainly his unparalleled goodness. But he who is good, envies none; therefore he produced all things like himself. This is the most rational motive for the creation of the world. For when God had resolved to make the world replete with every good, and to admit nothing evil, as far as the nature of things would permit, whatever came under his survey, though not tranquil nor at rest but extremely agitated and fluctuating,—those things he took and their chaotic mass he reduced to order; for this he conceived to be most eligible. It is not indeed, nor was it ever possible, that He who is the best of beings, should make any thing but what is most beautiful. When therefore he possessed intelligence himself, he found nothing destitute of intelligence better than what was intelligent, among those things which could be perceived through the whole range of existence; and that the fitness of things required intelligence not to be united to any being without mind. Wherefore he inclosed intelligence in the mind, and mind in the body. Thus he judged that the work, when completed, would be most beautiful. On this account we do not hesitate to affirm, (if conjecture can avail any thing in our investigation,) that this world is a living and intelligent being, formed by divine Providence. Granting this, it follows that we should ascertain what animated existence God selected as his model in the formation of the world. Certainly none of those living creatures which are known to us; for they are all subdivided into different species, or are incomplete, not being in every part perfect; but nothing can be beautiful which resembles what is imperfect or incomplete. We may therefore say, that the world resembles that form of things of which all animals are as it were certain particles, whether they be considered individually or as belonging to different species. All living creatures then which are conceivable in the mind, or which are subject to examination, are comprehended within the range of intelligence and reason; such as men and beasts and all things which in this world come under our inspection. And since the knowledge of what is most beautiful in the nature of things, and what is in every respect most complete, may be acquired, when it was the pleasure of God to form a world of a similar appearance, he made one living visible being in which all creatures might be contained. Have we now said correctly, that there is only one world? Or would it be nearer the truth and better to say, that there are many, or that there are worlds innumerable? There is surely only one, if it was made after this form: For what comprises all those living creatures which come within the reach of our knowledge, can have no association with another. It would be necessary again that there should be another animated world, in which this should be contained, the parts of which should be living beings of a superior class, and these heavens, which would be the resemblance not of what is nearest, but of what is now most distant. That nothing of this nature might happen, and that this world might be the perfect resemblance of a perfect form on this very account, because single and alone; therefore God made this world by itself, and gave it a distinct character of its own. Besides every thing produced must be corporeal, visible, and tangible. Nothing, however, which is destitute of fire can be seen, nor can any thing be touched which is without solidity; but there is nothing solid of which earth does not make a component part. For which reason, God designing to frame the world, first united earth and fire: But two principles require a third to make all cohere, and to serve as a link or bond. Now the finest and best-adapted bond is that which forms the most intimate union between itself and those things which it connects. What the Greeks call **ANALOGY** may be here applied; and if we must venture on the appropriation of terms, since we have been the first to introduce this subject, in Latin it may be called **RELATION** or **PROPORTION**. For when of three numbers or figures, or any description of things, the middle term as well as the first in a proportion, is compared with the last, and again the middle with the first and the last with the middle, the middle term may be sometimes the first and sometimes the last; but the extremes become means, by the force of necessity, that those things which have been disunited, may have some relation; and when relation has taken place, the consequence is, that all becomes one body. And if the body of the Universe were brought into existense, unvaried, uniform, and wanting nothing, one medium being furnished might connect itself and those things between which it was placed. But when solidity was required for the world, (and no solids can ever through one medium be made to cohere, two always being required,) so it happened, that God placed water and air between fire and earth, and established a proportion among them, that as fire was to air, so air to water, and that air should bear the same proportion to water as water bore to earth.

Idle. W. VINT.

ON THE SIEGE OF OSTEND.

A Translation from the Latin.

The city of Ostend is celebrated in History for the successful resistance which it made, for thirty nine months, against the flower of the Spanish army. This was at the period of the commencement of the liberties of the Low Countries, when all the country around Ostend fell into the hands of the Archduke Albert. During that long siege, the inhabitants constructed a new harbour, because the old one was blockaded by the enemy; and began to erect a new Town on some vacant ground. They fearlessly imported provisions and arms through the hostile ranks of the besiegers; and in this protracted conflict with the enemy, with disease, famine, and various inundations, which called into exercise all their activity and patience, they lost, it is said, a hundred and forty four thousand men. In the year 1604, being the fourth of the siege, Ostend, after a brave defence, fell into the hands of the Spanish General Spinola; but offered nothing worthy of the acceptance of the conquerors, except heaps of ruins, and a few human beings emaciated by hunger but in spirit unsubdued.

Ostend, small field with leaders overspread,
To friends an honour, and to foes a dread,
On which the civilised world looks down,
Far loftier by surrounding ills has grown.
She, who both Fate and ruin dare withstand,
Sits like an exile in a foreign land.
Thrice has the year pursued its wonted round,
A change of foes as often we have found;
Winter brought with it inundating seas,
And summer raged with pestilent disease:
Yet these were not the climax of our woes,
The arms of Spain have proved our greatest foes;
Disease its race amongst us has begun,
No death is single, and no bier alone.
Fortune, why linger? what reward pertains
To the possessor of these foul remains?
Why hold the shades of heroes wet with gore
From kindred spirits on the Stygian shore?
What dying man shall occupy these tombs,
Now that our foes have met their several dooms?
And is the battle only to be fought
On ground to barrenness and ruin brought?

London. John M. Hare.

THE VALLEY OF FERN.

From Poems by Mr. Bernard Barton, of the Society of Friends.

THERE is a lone valley, few charms can it number,
Compar'd with the lovely glens north of the Tweed;
No mountains enclose it where morning mists slumber,
And it never has echoed the shepherd's soft reed.
No streamlet of crystal, its rocky banks laving,
Flows through it, delighting the ear and the eye;
On its sides no proud forests, their foliage waving,
Meet the gales of the Autumn or Summer wind's sigh.
Yet by me it is priz'd, and full dearly I love it,
And oft my steps thither I pensively turn;
It has silence within, Heaven's proud arch above it,
And my fancy has nam'd it the Valley of Fern.

O deep the repose which its calm recess giveth!
And no music can equal its silence to me;
When broken, 'tis only to prove something liveth,
By the note of the sky-lark, or hum of the bee.
On its sides the green fern to the breeze gently bending,
With a few stunted trees, meet the wandering eye;
Or the furze and the broom their bright blossoms extending,
With the braken's soft verdure delightfully vie;—
These are all it can boast; yet, when Fancy is dreaming,
Her visions, which Poets can only discern,
Come crowding around, in unearthly light beaming,
And invest with bright beauty the Valley of Fern.

Sweet Valley! in seasons of grief and dejection,
I have sought in thy bosom a shelter from care;
And have found in my musings a bond of connexion
With thy landscape so peaceful, and all that was there:
In the verdure that sooth'd, in the flowers that brighten'd,
In the blackbird's soft note, in the hum of the bee,
I found something that lull'd, and insensibly lighten'd,
And felt grateful and tranquil while gazing on thee.
Yes! moments there are, when mute nature is willing
To teach, would proud man but be humble and learn;
When her sights and her sounds on the heart-strings are thrilling;
And this I have felt in the Valley of Fern.

For the bright chain of being, though widely extended.
Unites all its parts in one beautiful whole;
In which Grandeur and Grace are enchantingly blended,
Of which GOD is the Centre, the Light, and the Soul!
And holy the hope is, and sweet the sensation,
Which this feeling of union in solitude brings;

It gives silence a voice—and to calm contemplation
Unseals the pure fountain whence happiness springs.
Then Nature, most lov'd in her loneliest recesses,
Unveils her fair features, and softens her stern;
And spreads, like that Being who bounteously blesses,
For her votary a feast in the Valley of Fern.

And at times in its confines companionless straying,
Pure thoughts born in stillness have pass'd through my
mind;
And the spirit within, their blest impulse obeying,
Has soar'd from this world on the wings of the wind:—
The pure sky above, and the still scene around me,
To the eye which survey'd them, no clear image brought;
But my soul seem'd entranc'd in the vision which bound me,
As by magical spell, to the beings of thought!
And to HIM, their dread Author! the Fountain of Feeling!
I have bow'd while my heart seem'd within me to burn;
And my spirit contrited, for mercy appealing,
Has call'd on his name in the Valley of Fern.

Farewell, lovely Valley!—when Earth's silent bosom
Shall hold him who loves thee, thy beauties may live;—
And thy turf's em'rald tint, and thy broom's yellow blossom,
Unto loiterers like him soothing pleasure may give.
As brightly may morning, thy graces investing
With light and with life, wake thy inmates from sleep;
And as softly the moon, in still loveliness resting,
To gaze on thy charms, thy lone landscape may steep.
Then, should friend of the bard, who hath paid with his
praises
The pleasure thou'st yielded, e'er seek thy sojourn,
Should one tear for his sake fill the eye while it gazes,
It may fall, unreprov'd, in the Valley of Fern.

NATIONAL VIRTUE OF THE SWEDES.

"During this journey, the daily opportunities we had of remarking the honesty and simplicity of the *Swedes* were too numerous, and too striking, to leave any doubt upon our minds, as to the truth of the remarks we have before made respecting their national character. The most trivial incidents would sometimes afford striking traits of the disposition of the people, We shall mention one that occurred at *Norralo,* where we changed horses. In paying the driver the usual paper-money for the last post, there remained something due to us.

We told the man to keep this for his own use; and were driving off; when he ran after the waggon, bawling, that we had not received the change that was due to us. We made the interpreter explain to him, better than we had done, that, as he had so well deserved it, he might take the change for himself. "I understood the gentleman," said he, somewhat impatiently; "but is it not fitting that I should first give them what is due to them? and then, if they think proper to bestow any thing upon me, they may act as they please." The smallest donation not only satisfies the *Swedish* drivers, but rejoices them; and as an expression of their gratitude, they generally endeavour to kiss the hand of those from whom they receive any bounty at parting. It is among this people that robbery and murder are almost unknown. In the various opportunities of pilfering from a traveller, offering temptations to theft, which are rarely resisted in other countries, no instance occurred of their taking any thing belonging to us, or in any way attempting to defraud us."

"Near the road from Orebro, in Sweden, there commonly occurred upright posts, supporting boxes for receiving charitable donations: these had generally a small shed placed over the box, and beneath the shed there was sometimes a picture representing the figure of a mendicant in the attitude of supplicating alms. We could but consider these little depots as so many monuments ot the honesty of the people: There is not any part of our own country, where, if alms were thus collected, the boxes for containing them would remain safe from violation in the public high-ways during a single night. Another proof, whether of good government or of great virtue, in *Sweden*, is, that high-way robberies are unheard of. No one thinks of guarding against an evil which is never experienced; therefore the traveller proceeds on his journey unarmed, and in perfect safety, at all hours of the day and night; neither is his property liable to the attacks of pilferers, in places where he may happen to rest: not an article would be stolen from his carriage, if left in the public street or road; whereas in *Russia*, every bit of the harness and tackle would be carried off, every moveable thing purloined, and bolts and bars be found insufficient to protect whatever effects he may have carefully locked within his trunks."

Dr. E. D. Clarke's TRAVELS.

ENTERPRIZE OF THE ITALIAN PICTURE VENDERS.

Being conducted to an inn, we found here an Italian, a native of *Como* in the *Milanesse* territory, who was travelling with seven carts, containing about forty Wax Figures, for exhibition, as large as life. He told us, that, during the last week, in the little town of *Hernosand*, (in Sweden) upwards of four hundred persons came to see his exhibition, at a shilling *English* for each person: a large sum in these parts.—How remarkable are the industry and persevering enterprise of the natives of *Como;* a people wandering all over the earth during their youth; and, at the approach of old age, generally returning home, rich with the fruits of their ingenuity and labour! Nineteen out of twenty of the vagrant Italians that appear in any part of *Europe*, are from the lake and territory of *Como*. In *England*, they carry heavy baskets, filled with barometers, thermometers and cheap coloured engravings, framed and glazed, of Scriptural subjects: With regard to those who exhibit wax-work, as we found one of them upon the borders of *Lapland*, so it may be remembered Mr. *Walpole* mentions *Campioni*, at *Constantinople*, announciug his arrival in modern *Greek*, and informing the inhabitants of *Pera*, that he had brought with him "forty Figures of the Kings of *Europe*, and other illustrious personages, all of the size of nature."

Idem.

The Furniture of a Norwegian Dwelling, and the Fall of Superstition.

The next day, *October* 11*th*, (at Ronne in Norway,) they brought in a breakfast worthy of "the land of cakes:" we had coffee, and tea, and brown bread, and butter, and eggs, and the sort of cheese, scraped, which we before mentioned as being seasoned with sugar. The walls of the apartment consisted of bare timber, without any kind of covering: but the cupboards, the beaufets, chairs, and window frames, were all painted, and the windows had large squares of glass. The inside of such a room affords no bad criterion by which to judge of the progress of refinement and the state of things, in the country: wretched engraved portraits, daubed with the most glaring colours, hung about the walls, together with small mirrors in gaudy lackered leaden frames, the tawdry manufacture of those wandering *Italians*, the natives of *Como* in the *Milanese* territory, who hawk these wares all over Europe. A kind of ornamental hanging of very coarse gauze is fastened over the windows in these houses, falling down on each side: it is placed for no other purpose than to make a display of finery, and of the tidiness of the housewife, in keeping it starched and clean. The rooms are well floored; and the ceiling, although of deal, is cased with planks neatly wrought. In the corner of these apartments, as we still see it often in *England*, there is generally placed a *beaufaîte*, or *beaufet*,[*] with a glass door, containing their little stock of plate: a few spoons, a pair of sugar-tongs, a sugar pot, with some figure glasses, and old *China* cups and saucers. *English* earthenware begins to find a place among these rarities; coffee-pots, and punch-bowls, and mugs, with pictured representations of jolly tars and their mistresses, inscribed with ENGLISH patriotic or convivial sentiments, or exhibiting views of sportsmen sallying forth in cocked hats to the chase, reminded us of the old fashions of our *terra-cotta* manufacturers. But that which more forcibly struck us, as a curious relique of past times, was the *Censer*, or *Incense Pot*, of a *Roman Catholic* Church, suspended near the stove, to supply the place of a chaffing-dish for lighting tobacco pipes. The people of the house said, they had procured this *Censer* from an old Church which formerly stood below, the village upon the shore of, the *Miosen*. It spoke volumes to us; because it was impossible to view this once proud symbol of priesthood without connecting its fallen dignity with the sure fate of superstition. The antiquity of the rite for which it served, almost entitles it to reverence. The first mention of a *Censer* in history occurs in the oldest book of the world: the two sons of AARON "took either of them his *Censer*, and put fire therein, and put *Incense* thereon, and offered strange fire before the LORD."—The same custom of burning incense in the Temple, which was ordained as "perpetual through all the generations" of the holy men of LEVI, was also common among the *Heathen* nations. The *Roman Catholics*, who have preserved many Heathen customs and superstitions, from which our own Church is not altogether free, continued the ancient practice of burning incense; and have made the observance of it essential to their most sacred rites; preserving even in the form of their *Censers* the customs of the earliest ages. Whosoever therefore, upon a single theme, would concentrate the whole of an enquiry calculated to illustrate the rise and progress of Superstition, and to develope the gradual working of REVELATION towards the final overthrow of this many-headed monster, may write "*the Adventures of a Censer*," consigned, after all the periods of its splendor, to an obscure Inn, in an obscure village, of this remote country: and telling in its degraded state, the inevitable destiny of BABYLON, who hath "MADE ALL NATIONS DRINK OF THE WINE OF THE WRATH OF HER FORNICATION."

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[*] Hence, perhaps, our word *Beef-eater*, as applied to servants of the crown, or Beaufaitiere, a waiter at the Beaufet.

Idem.

ON WIT AND HUMOUR.

In the following extracts from the late Professor Millar's "Essay on Dramatic Poetry," the reader will find some incorrect positions amid many ingenious remarks and much entertainment. No great deference can be paid to his observations on the ancient Greek Drama, the chaste comedies of which he represents as *mere farces*. It was his misfortune that he had not become acquainted with them in the beautiful language in which they were originally composed, and in which many of their charms are concealed. He had only perused them in the garb of a translation; and his accusation of *tameness* will hold equally good when applied to the ancient EPIGRAMS, which very rarely contained any thing resembling the PUNS of modern Epigrams, but which are nevertheless admired by men of taste and discernment, for the delicacy of their expressions and the point of their language.

"It also merits attention, that the same varieties in character and situation, which furnish the materials of humour and ridicule, dispose mankind to employ them for the purpose of exciting mirth. The standard of dignity and propriety is different according to the character of the man who holds it, and is therefore contrasted with different improprieties and foibles.—Every person, though he may not be so conceited as to consider himself in the light of a perfect model, is apt to be diverted with the apparent oddity of that behaviour which is very different from his own. Men of robust professions, the smith, the mason, and the carpenter, are apt to break their jests upon the weakness and effeminacy of the barber, the weaver or the tailor. The poet, or the philosopher in his garret, condemns the patient industry and the sordid pursuits of the merchant. The silent, mysterious practitioner in physic, is apt to smile at the no less formal but clamorous ostentation of the barrister. The genteel military man, who is hired at the nod of his superior, to drive his fellow-creatures out of this world, is ready to sneer at the zeal and starch-deportment of the Divine, whose profession leads him to provide for their condition and enjoyments in the next. The peculiarities of each individual are thus beheld through a mirror, which magnifies their ludicrous features, and by continually exciting that 'itching to deride,' of which all mankind are possessed, affords constant exercise to their humorous talents."

He then applies this principle of the subdivision of professions to the comic compositions of different nations.

"As in the most commercial of the Greek states, almost all the departments of trade and manufactures, and even many of those which in modern times are accounted liberal, were filled with slaves, the uniformity of character so prevalent in that class of men, was, in a great measure extended to the whole body of people, and produced a proportional deficiency of those objects which afford the chief materials, as well as the chief excitements of humour and ridicule. This was probably the reason why the Athenians, notwithstanding their eminence in all the other productions of genius, discover so remarkable a deficiency in comic or ludicrous compositions. The comedies of Aristophanes, written at a period when the nation had attained a high pitch of civilization, are mere farces, deriving the whole of their pleasantry, not from nicely discriminated and well-supported characters, but from the droll and extravagant situations in which the persons of the drama are exhibited. It is true, that the style of what is called the *new comedy*, is said to have been very different; but of this we can form no judgment, unless by the translations or imitations of it by Plautus and Terence; from which the originals, in the article which we are now considering, do not appear in a very favourable light.

"The comedies of those two Roman writers are also very different in the representation of character. An old avaricious father, a dissolute extravagant son, a flattering parasite, a bragging cowardly soldier, a cunning intriguing rascal of a slave; these, with a few trifling variations, make the *dramatis personæ* in all the different compositions of those authors. But though neither Plautus nor Terence appears to have much merit in describing those nice combinations of affectation and folly, which may be regarded as the foundation of true comedy, they seem happy in the expression of common feelings, and in exhibiting natural pictures of ordinary life.

"The Romans, independent of their close imitation of the Greeks, had scarcely any comic writing of their own. After the destruction of the commonwealth, we meet with few writers in this department; and none of any eminence. The age of elegant literature at Rome was very short: there was no commerce: the number of slaves was immense, as no free citizen would engage in any profession but those of the camp or the bar; and therefore it is probable that the Romans were still more deficient than the Greeks in that variety of original characters which is the great spur to ridicule.

"In modern Italy, the rise of mercantile towns was followed by the revival of letters, and by the introduction of ludicrous and somewhat licentious compositions; but the Italians lost their trade, and their literature began to decline before it had risen to that height at which the improvement of comedy was to be expected. They displayed, however, in a sort of pantomimic entertainments, a vein of low humour, by grotesque exhibitions, which are supposed to characterise the citizens of different states; and in this inferior species of drama, they are said to possess irresistible powers of exciting laughter.

"In France, the country which after Italy made the first advances in civilization, the state of society has never been very favourable to humorous representation. In that country, fashion has had more influence than in any other part of Europe, to suppress the oddities and eccentricities of individuals. The gentry, by their frequent intercourse, are induced to model their behaviour according to a common standard; and the lower orders think it incumbent upon them to imitate the gentry. Thus a great degree of uniformity of character and behaviour is propagated through all ranks, from the highest to the lowest; and a French beggar is a gentleman in rags. Individuals, at this rate, have little temptation to laugh at each other; for this would be nearly the same thing as to laugh at themselves. From refinement of manners, at the same time, their attention has been directed to elegant sallies of pleasantry, more than to ludicrous and buffoonish representation; and the nation has at length come to occupy the superior regions of wit, without passing through the thicker and more vulgar medium of humour.

"It may, accordingly, be remarked, that among the numerous and distinguished men of genius whom France has produced, Le Sage, and Moliere, are perhaps the only examples that can be adduced of eminent humorous writers. The high and deserved reputation of the latter as a writer of comedy, is universally admitted; though I think it can hardly be denied, that his characters are commonly overcharged and farcical.

"There is, perhaps, no country in which manufactures and commerce have been so far extended as in England, or consequently in which the inhabitants have displayed such a multiplicity and diversity of characters. What is called a *humourist*, that is, a person who exhibits particular whims and oddities, not for the sake of producing mirth, but to gratify his own inclination, is less known in any other country. The English are regarded by their neighbours as a nation of humourists; a set of originals, moulded into singular shapes, and as unlike the rest of mankind as each other.

* * *

"Political reasoners have ascribed this wonderful diversity of character among the English to the form of their governmrnt, which imposes but few restraints upon their conduct. It is obvious, however, that, though an absolute government may prevent any great singularity of behaviour, a free constitution will not alone produce it. Men do not acquire an odd or whimsical character, because they are at liberty to do so, but because they have propensities which lead them to it. In the republican states of antiquity, which enjoyed more political freedom, and among mere savages, who are almost under no government at all, nothing of this remarkable eccentricity is to be observed.

"But whatever be the cause of that endless diversity of characters which prevails in England, it certainly gives encouragement to sarcastic mirth and drollery; and has produced a general disposition to humour and raillery, which is the more conspicuous from the natural modesty, reserve, and taciturnity of the people. To delineate the most unaccountable and strange appearances of human nature, they require not the aid of fiction; to conceive what is ridiculous, they have only to observe it. Each individual, according to the expression of a famous buffoon, is not only humourous in himself, but the cause of humour in other men. The national genius, as might be expected, has been moulded and directed by these peculiar circumstances, and has produced a greater number of eminent writers, in all the branches of comic and ludicrous composition, than are to be found in any other country. To pass over the extraordinary genius of Shakspeare, in this, as well as in other departments, with those other comic writers who lived about the commencement of English manufactures, and to mention only a few instances, near our own times; it will be difficult for any country, at one period, to match the severe and pointed irony of Swift; the lighter, but more laughable satire of Arbuthnot; the gentle raillery of Gay; the ludicrous and natural, though coarse, representations of low life by Fielding; the strong delineations of character, together with the appropriate easy dialogue of Van Burgh; the rich vein of correct pleasantry, in ridiculing the varieties of studied affectation, displayed by Congreve; and, above all, the universal, equable, and creative humour of Addison."

BARLOW ON MAGNETIC ATTRACTIONS.

The variation of the compass, a subject at all times no less interesting to the philosopher, than useful to the navigator, was first discovered, we are told, by Columbus, in his voyage to America in 1492; and Professor Gillebrand of Gresham College, in 1625, ascertained that this variation was itself of a changeable nature. The discovery of these important and very extraordinary facts, excited a lively interest, among men of learning and science, throughout all Europe; the exertions of our celebrated countryman, Dr. Halley, on this subject, are well known, and need not be here repeated.

The phenomenon of the magnetic dip, or inclination of the needle, accidentally discovered by Norman in 1592, was also then a subject of much speculation and inquiry; and, to render this law in the magnetic system subservient to science and navigation, the latitude, in any given meridian was attempted to be ascertained by its results; but, the delicacy of the instrument, and experience, very soon proved its demonstrations erroneous; and, until the last voyage of captain Flinders, was adverted to more as a matter of curiosity to philosophers, than of utility to navigators.

The diurnal variation of the compass, first discovered by Mr. Graham, who has been followed by Mr. Weirgentin, Mr. Canton, and, last of all, the indefatigable exertions of Colonel Beaufoy, likewise excited considerable attention; but though numerous theories have been formed to account for this phenomenon, none, as yet, have appeared satisfactory to philosophers, or useful to science, if we except Mr. Barlow's theory, which we here intend shortly to notice.

From the beginning of the 18th, to the 19th century, this very important, and highly useful, branch of science, was allowed to slumber; since nothing of the least consequence, during that long and enlightened period, was added to our previous knowledge, on this interesting subject. It is, indeed, true, that several distinguished navigators observed, during their respective voyages, anomalies in the variation of the compass, altogether inexplicable; and, what appears very extraordinary, the more pains that was taken by them to discover them, the further did they go away in point of theory.

They all found the errors of variation, without being able, in any way whatever, to account for them; until Captain Flinders, that acute and penetrating, but unfortunate man, in his last voyage of discovery, to Terra Australis, in 1801, 1802, and 1803, first discovered the true cause producing these hitherto unaccountable discordances in the variation on shipboard—a change in the direction of the ship's head. Having ascertained this most important truth, it soon occurred to him, that a local attraction must exist in the ship; which, in connexion with terrestrial magnetic attraction, acted on the magnetic needle, when placed at the binnacle, with a compound force; and, therefore, he found by experiments, that in the northern hemisphere, when the head was at *west*, this combined attractive power drew the north end of the needle to the *west*,; and in the southern hemisphere, to the *east*, of the true magnetic meridian: This difference, produced by local attraction, he denominated the '*Deviation*.'

Finding the maximum deviation in both hemispheres, when the ship's head was at *west* or *east*; and that the needle stood right when the head was in a direction with the magnetic meridian—north and south;—"What was the proportion of deviation," he asked himself, "at the intermediate points, between the *east* and *west* and *magnetic meridian*?" After much labour and consideration, it appeared to him, "that the errors produced by local attraction should be proportionate to the *sines* of the *angles* between the ship's head and the magnetic meridian; and, therefore, in order to find this proportion, it seemed probable the following *Rule* would be found applicable in all parts of the world, viz. "that the *error produced at any direction of the ship's head, would be to the error at east or west, at the same dip, as the sine of the angle between the ship's head and magnetic meridian was to the sine of eight points, or radius.*"

Captain Flinders dying soon after his return to England, we see no further attempts made either to verify or overthrow the accuracy of this *rule*, until 1807, when a small practical work, entitled, "An Essay on the Variation of the Compass, by W. Bain, Master in the royal navy," was published. In this valuable tract the author, in the last section, has, in a variety of examples wholly incompatible with the supposition of truth, completely exposed the fallacy of this rule—which Captain Sabine fully corroborates by observations made during the late arctic expedition.

Mr. Barlow, author of the work before us, an able mathematician, and one of the Professors of the Royal Military Academy, Woolwich, sensible of how very much real importance a formula, founded on correct principles, for correcting the deviation produced by a change in the direction of the ship's head, in all approachable latitudes, would be to science and navigation, and, indeed, to mankind in general, has at length arrived at the conclusion, after a long, laborious, and patient investigation of the laws of magnetic attraction, which his situation and place afforded the most ample opportunity and means for experiments, no less honourable to himself than beneficial to science, and practical navigation. Before we introduce our readers into our author's workshop, it may be proper they should clearly understand what Mr. Barlow's ideas were, on this subject, before he commenced operations; and the theory on which all his future hopes depended. We shall transcribe his own words:—"Since the iron of the vessel," says our author, "and the compass must be supposed to maintain the same relative position, with respect to each other, during the voyage, I imagine it to be possible to place a single ball of iron, equal to the whole mass, in a certain situation of the ship, when its effects upon the needle would be the same as that of the iron in its distributed state; or what amounts to the same thing, that all the forces acting on the needle in the actual state of the iron, may be reduced to a single resultant. I then assume, that a less mass of iron (having its entire attraction, or resultant, in the same line as the former,) may be approximated so near to the compass as to produce an effect equal to that of the iron of the vessel, whereby the tangent of the angle of deviation may be at any time doubled, and hence the deviation itself determined. Under this point of view, however, a slight computation would be requisite; but since the tangents of small arcs have very near the same ratio of the arcs themselves, we may suppose the angle itself doubled by the experiment, and hence the deviation ascertained by observation only."

Though our author's first experiments rather involved him into difficulties, at least proved nothing at the time, yet it is necessary the reader should know the apparatus he worked with, as well as the method by which he worked with it:—"I began," he says, "by describing, on a platform, several concentric circles, from eight to sixteen inches radius, drawing through the centre a line, in the direction of the magnetic meridian; I then set off my east and west points; and, lastly, divided the whole circle into equal parts of 10° each." With the compass over the centre of these concentric circles, he passed round successively, on each circle, several shells of different diameter; which, as our author anticipated, produced results at the time wholly inexplicable.

At length, however, our author, having exchanged his platform for a strong table, divided according to the points of the compass, with a circular hole of ten inches diameter in the centre, through which a ten inch shell was, at pleasure, made to pass by means of a block and pully, he again commenced operations, by passing the compass on the circle round the ball, instead, as before, the ball round the compass; "I elevated the ball," he continues, "till its action was imperceptible; and then gradually lowering it, I noticed the deviation at various altitudes of the ball, with the compass of each point of division on the circles; observing also very accurately the height or depth of the centre of the ball above or below the pivot of the needle, when the deviation was zero. These results, indeed, were the only ones applicable to my present inquiry; and from them I ascertained that the several parts of no action were all situated in one plane; the inclination of the plane itself to the horizon being found nearly equal to 20°, declining directly from the magnetic north point to the south. *This plan is, therefore, either exactly or nearly perpendicular to the direction of the dipping needle.*

The different manifestations indicated in these important experiments, between the needle and the attractive power of the ball, at different angles, horizontally and perpendicularly, induced our author to believe, that there were in every ball of iron two planes, in which the compass may be any where posited, without being influenced in its direction; the one that of no attraction, and the other the vertical plane, corresponding to the magnetic meridian. In consequence of which, he conceived an ideal sphere to be circumscribed about the ball of iron; and assuming the circle of no attraction as an equator, and the poles of that circle as the poles of the sphere, he imagined circles of latitude and longitude to be described round the ball in several circles, keeping it always at the same distance from the centre; and, therefore, when these ideal circles of latitude and longitude were correctly ascertained by calculation, and verified by experiment, as particularly described in page 24, and to which we must refer our readers, he succeeded in establishing the law of deviation as it respects the latitude; namely, *that the tangents of the deviations are proportional to the rectangle of the sine and cosine of the latitude; or to the sine of the double latitude, which is the same thing.* In the same way our author establishes a like law of elevation, as it respects the longitude; and, to explain himself more fully, he has given two diagrams, one describing the laws of deviation as it regards the latitude, and the other as it regards the longitude.

With regard to the laws of attraction as it regards the mass of iron, our author, has by a great variety of experiments, verified by computation, clearly demonstrated, that the power of attraction resides wholly on the surface, and is independent of the mass; or, in other words, *that the tangents of the deviations are proportional to the cubes of the diameter, or as the* $\frac{3}{2}$ *power of the surface, whatever may be the weight or thickness.*

The striking confirmation of the existence of the plane of no attraction in the most irregular masses of iron; and that the power of attraction resides wholly on the surface, and is independent of the mass; as exhibited by experiment, and confirmed by computation, must have gratified and encouraged our author in his toilsome and unbeaten path; and, therefore, to put the matter beyond all dispute, he determined to verify his former experiments by others of a different nature, and on a much larger scale.

Our author procured for his next experiment, an iron 24 pounder, mounted on a platform which admitted of its being traversed through an entire circumference; the tracks at the bottom running over a circle ten feet six inches diameter, divided into 32 equal parts corresponding with the points of the compass. A piece of wood, projecting four feet from the muzzle of the gun, for the compass to stand on, was made to fit exactly the bore of the gun, on which the compass could be moved to any distance, at the time of experiment. As it is impossible to make room for the results, we shall just now say, that they fully corroborated the accuracy of the results given by former experiments; and that, in the present instance, the difference between the observed and the computed results it so very trifling, as to be almost imperceptible.

Our author has applied his rule in the computations of deviations found by Captain Sabine on board the Isabella, off Shetland, where the dip was, 74° 21′, with great success; since the greatest error between Captain Sabine's observed deviation, and the computations of our author, only amounts to 49′.

From the singular discovery, "that the power of attraction resides wholly in the surface of iron bodies, and is independent of the mass," our author, by one of those happy ideas so peculiar to genius, conceived the possibility of ascertaining the correct deviation in all ships, in all positions, and in all places, by simple observation only, and independent of computation. With this view, he ordered a frame-work to be affixed to the gun, which should project beyond the compass, whereby he could suspend a ten inch shell in any required position with respect to the centre of the needle. Having fixed the ball in the required situation, he repeated his first course of experiments, which we have already noticed, with the ball attached, by traversing the gun through the entire circle.

From the results indicated in the table which he gives, our author assumes, and with some degree of probability, that when the whole quantity of deviation is once ascertained by swinging the vessel, the navigator by attaching a plate of iron to the binnacle in a plane with the centre of greatest deviation, or local attraction, which, he says, will be found in most vessels at an angle between 20° and 60° may at all times, and in all places, find, without computation, the exact deviation, by turning round the binnacle, with the plate of iron attached. In like manner our author made his experimental results inserted in the table: Or, in his own words, "supposing now this first approximate angle to have been found, then the plate must be fixed, so that its centre of attraction inclines from the pivot of the compass, at about the same angle; then turning round the binnacle, point by point, observe the deviation caused by the plate; and if these correspond with those given by the vessel, the plate is rightly adjusted; but if not, (as is most likely to happen,) such trifling changes may be made in its position, which will be attained in a few trials." Our author has given a particular description of this plate, and the most proper method of attaching it permanently to the binnacle. We must reluctantly decline entering, as we fully intended, upon our author's beautiful theory, by which he determines the laws producing the diurnal variation of the compass, which, from the analogy of experimental and computed results, appears unquestionable; and which, being bottomed on correct philosophical principles, seems to us the only theory deserving of the least attention. *Blackwood's Magazine.*

Living Toads found in Stones are productions of the former world.—By the Rector of Pabsdorf.

The occurrence of living toads in stones, is one of the most remarkable facts in natural history. Amongst many examples of this sort, we shall mention a few which put the matter beyond all doubt. A living toad was found in a large stone, at Newark on Trent, in England. It was of a white colour, measured three and a half inches, but appeared incapable any more of bearing the light. For all its motions argued an incompatible state, and an hour afterwards it died. But in this time it was seen by several hundred people.

In a stone quarry, near Cassel, the workmen discovered three living toads lying together in a stone four feet long, three feet broad, and as many high, on the outside of which, before it was broken, not the slightest trace of an aperture was to be discovered. It was with difficulty that these animals could be brought from the spot they lay in, and as soon as they were taken out, they hopped in again. They appeared at first to be quite lively in the grass; but they died in half an hour.

The fact cannot, therefore, be disputed, and I could, were it necessary to prove the truth of these appearances, quote many instances of this sort, which have been recorded. Some time since a living toad was found in slate, at Rothenberg on the Saal. We shall not, therefore, detain ourselves longer on this point, but endeavour rather to explain the matter.—Every thinking reader, who has not heard of this phenomenon, will consider such as wonderful, and many even unaccountable. It appears also at first sight to be impossible, for a creature to be inclosed in a stone, such a length of time, without dying of hunger, or being suffocated.

Naturalists have endeavoured, to be sure, to shew, how this is possible; but no one has, if I remember, explained in what manner and when these animals came into the stones.

In order to solve the first problem, it is said, the stone in which the toads existed, was probably a porous sand-stone, which imbibed moisture from rain, which the animal inspired by means of its pores, or its sucking warts. For these animals can be kept long alive on wet blotting paper, which is moistened from time to time. It is also known that toads and frogs are very tenacious of life, and can fast a long time.

An English naturalist made a trial, how long he could keep a toad without nourishment; he placed it in a pot, and buried it in the ground, closing it carefully. He forgot by chance to dig up the pot, until two or three years were elapsed. He found his toad still living, and buried it afresh. We have to wait the issue.

But this explanation does not appear quite satisfactory to us. Such a creature can be preserved living by means of moisture or water, for a certain time.—But many thousand years, how would that be possible? For we cannot admit of a shorter period, since which our rocks, even slate, lime, and sandstone, and who knows, even if it were a porous sandstone in which the toads lived.

We can more easily explain how such an animal can exist and be preserved in a tree. For a living toad has been found in the cavity of a tree, which, according to its rings, must have been more than eighty years old. It probably had crept into a hole of one of its boughs, and had not been able to come out again; and the opening had in the course of time completely closed. Here it could easier subsist, than in hard stone, but the sequel will show, that the preservation of these animals does not depend upon nourishment, but upon another circumstance, and quite other causes. We come now to the second question, how and when the toads came into the stones. In order to render this clear to ourselves, we must remember, that besides our own present world, in which we exist, one has already preceded it, which contained, as ours, terrestrial and marine animals.—Yet there was a time, when the whole continent was but an immeasurable ocean; as the secondary mountains, with their petrified beds of muscles, fishes, and sea productions prove. After some unknown great catastrophe, which our earth suffered, the sea at length disappeared, and from *a world of waters* arose, if I may be allowed the expression, *a world of land.*—There, where at present the plough turns up the soil, and countless corn fields shine with their golden harvests, where immense forests spread forth their luxuriant trees, amongst which numerous wild animals sport, where hills and mountains raise their varied summits, where herds of cattle graze, where rivulets and rapid streams wind through the vallies, and where cities and villages are now situated, there formerly raged the waves of this ocean—there swarmed hosts of animals, of numberless forms and magnitudes.

At the command of the Almighty the waters disappeared, and with them the then existing world of marine animals and of plants, which were thus placed upon the dry land.

The bowels of the earth have preserved to our times the remains of such only as have withstood decay, and have become petrified. And the bottom of the sea became dry land, and the slime and mud it had left behind was hardened into stone. But another terrestrial world, besides the one of water above mentioned, must have existed, before the present one was formed. This can be seen from the numerous remains of terrestrial animals and productions which we find in different countries, and which do not belong to the present period of the earth. There are as many and as large forests under the earth as there are above it, which have been buried thousands of years ago, and have been transformed into coal. There were formerly as many, perhaps more, large and small animals on the earth than there are at present. We must, therefore, suppose that the sea and dry land have been continually changing places with each other on the surface of our earth, and that after each change of this description a new creation of animals and plants took place on it. For this reason we find, that wood in a state of coal, and the bones of quadrupeds, occur intermixed with marine productions in the same bed; nay, even under the bottom of the sea we discover river muscles, and the beds of former great rivers. It may be conjectured, that at a future transformation of the earth new intermixtures will arise, and the productions of our present world will be united to those of a former one, and rest with them in one common grave, in order to make place for a new and better world. It is impossible to determine the time when the last great transformation took place, which caused the former world to make place for this. But every one who knows how much time is necessary to produce a new creation of plants and animals out of the bosom of the earth, according to the laws of nature, must easily discern that many centuries must have passed away since that great catrastrophe happened.

The living toads already mentioned must have been inclosed in their stony prisons during this last revolution of the globe. For on the present period of the earth having commenced, and the productions of the former world being buried in mud and slime by the overflowing of the sea, the whole surface of the earth became turned into solid strata by some unknown process of nature, and out of the sand-banks and coral reefs of the sea, arose the secondary limestone and sand stone mountains. The toads of the former world met with the same fate as its fish and other animals: they were covered and buried with mud. They would have perished like their fellow creatures, in water or in mud, had not their peculiar organization prevented this. These animals possess the property of sleeping and remaining in a state of torpor during the winter, without having occasion for any nourishment during the whole period. Frogs are often to be found, in winter, in ice, and on its thawing, they are again revived. And it is well known, that frogs and toads, when the weather is warmer than usual in the spring, come forth from their holes in the earth, and commence a new life.—During the great revolution of our globe, just mentioned, when the whole animal and vegetable creation was buried under mud and earth; these toads met with a similar fate, and were inclosed in their stony prisons until they were released from them by accident. They were obliged to repose in them some thousand years in a state of sleep, having no other means in their power, otherwise they would have had a like fate with millions of fishes and terrestrial animals, which perished and became petrified.

(To be continued.)

ANSWERS

TO THE

QUESTIONS FOR YOUTH,

Proposed in the First Number of the Third Volume.

(1) *By Master C.* Bartholomew, *Woodhouse Grove.*

The sum of two numbers is 20, and the sum of their squares is 272; it is required to find them by a simple equation.

Answered by Master W. Pickup.

Let the numbers be represented by x and y; then $x + y = 20$, and $x^2 + y^2 = 272$; from the square of the first equation, subtract the second and we have $2xy = 400 - 272 = 128$; this subtracted from the second, gives $x^2 - 2xy + y^2 = 272 - 128 = 144$, therefore $x - y = \sqrt{144} = 12$; hence by adding and subtracting this from the first and dividing by 2, we have, $x = 16$, and $y = 4$, the numbers required.

Answered by Master R. Taylor.

Let $x =$ the difference of the two numbers; then will $\frac{20 + x}{2}$ and $\frac{20 - x}{2}$ express the two numbers.

Then per question $\left(\frac{20 + x}{2}\right)^2 + \left(\frac{20 - x}{2}\right)^2 = 272.$

or, $400 + x^2 = 544$. therefore $x^2 = 544 - 400, =$

144 or $x = 12$. Therefore the numbes are $\left(\frac{20 + x}{2}\right) =$ 19 and $\left(\frac{20 - x}{2}\right) = 4$ as required.

(2) *By Master B. SCHWARTZENBROOK.*

Required the side of a Tetrahedron; the solidity of which is equal to that of a cube whose side is 2.

Answered by Master R. MOUNSEY.

Put $x =$ the side of the Tetrahedron; then by Mensuration $\frac{x^3}{12}\sqrt{2} = 2^3$; Hence $x = \sqrt[6]{4608} = 4{,}0793$. The side required.

Again by Master *JOEL LIVSEY.*

Put $x =$ the side and $a = 2$; then $\frac{x^3}{12} \times \sqrt{2} = a^3$ per question, and $x = \left(\frac{12\,a^3}{\sqrt{2}}\right)^{\frac{1}{3}} = 4.0793$.

Otherwise by Masters *JOHN HARE, CHARLES MACAULEY and W. VINT.*

Let $x =$ the side required, then the perpendicular of one of the triangular surfaces will be $\sqrt{\frac{3x^2}{4}}$ Construct a triangle of this perpendicular and the perpendicular of the figure together with the segment between them at their extremities for a base, and the perpendicular of the figure will be found to be $\sqrt{\frac{2x^2}{3}}$ which multiplied by the area of its base will produce $\sqrt{\frac{x^6}{8}} = 24$ the content of a solid three times the solidity of the given figure; then $\frac{x^6}{8} = 576$, $x^6 = 4608$, $x = 4.0793$, *Ansr.*

(3) *By Master* MUDGE.

If you and I sit in the same room at the distance of 5 and 10 feet respectively from the fire, how much hotter is your seat than mine?

Answered by Master W. VINT.

As heat radiates in conical pencils of rays, supposing the diameter of the base of a cone the perpendicular of which is 5 feet to be 1 the diameter will be 2 when the perpendicular is 10 feet, and the area of one base will be to the other in the proportion of 1 to 4 the rays of heat diverging and occupying at the distance of 10 feet four times the space which they occupied at the distance of 5, and consequently the heat is in this proportion.

Again by Masters S. WALKER and R. TAYLOR.

The heat received by the two persons is inversely as the square of their distances from the fire, then as $5^2 : 10^2 :: 1 : 4$ that is, he that is the nearest the fire will receive 4 times as much heat as the other.

(4) *By Master* W. SWIFT.

The difference of two numbers is 2, and the difference of their fifth powers is 42242. Shew how to find them independently of completing the square, by a new method never before published.

Answered by Master W. HEBBLETHWAITE.

Let $x+1$ and $x-1$ represent the required numbers. Then by the question $(x+1)^5 - (x-1)^5 = 42242$, or, $10x^4 + 20x^2 + 2 = 42242$, and by addition $10x^4 + 20x^2 + 10 = 42250$, then $x^4 + 2x^2 + 1 = 4225$ by division, therefore $x^2 + 1 = \sqrt{4225} = 65$ by evolution, consequently $x = \sqrt{65 - 1} = 8$ by evolution, Hence $x + 1 = 8 + 1 = 9$ the greater number; and $x - 1 = 8 - 1 = 7$ the lesser number.

Again by Master C. BARTHOLOMEW.

Let x and y denote the numbers, then per question

$x - y = 2$ (1) and $x^5 - y^5 = 42242$ (2) Divide the second by the first and we have $x^4 + x^3y + x^2y^2 + xy^2 + yx^3 + y^4 = 21121$ (3) Add the fourth power of the first equation to 4 times the third, there results $5x^4 + 10 x^2y^2 + 5 y^4 = 84500$. Divide it by 5, $x^4 + 2 x^2y^2 + y^4 = 16900$. Extract the □ root $x^2 + y^2 = 130$. (4) From the fourth take the square of the first, then $2xy = 126$ (5) and by adding the fourth and fifth together, $x^2 + 2xy + y^2 = 256$. Extract the □ root $x + y = 16$ and $x - y = 2$. Ergo by addition $2x = 18$, or $x = 9$, and by subtraction $2y = 14$, or $y = 7$ the numbers required.

(5) *By Master* G. ASPDEN.

The area of a right angled triangle whose sides are in arithmetical progression is $216\sqrt{3}$. What are its sides?

Answered by Masters W. PICKUP *and* W. HAREWOOD.

Since the sides are as 3, 4 and 5, let them be denoted by $3x$, $4x$ and $5x$; Then the area $= \dfrac{3x \times 4x}{2} = 6x^2 = 216\sqrt{3}$ by Question. Therefore $3x = 18\sqrt[4]{3}$, $4x = 24\sqrt[4]{3}$ and $5x = 30\sqrt[4]{3}$, the respective sides required.

Again by Master C. MACAULEY.

Let the sides be denoted by $x - y$, x, and xy, then $x^2 - xy = 432\sqrt{3}$ = double the area and $2x^2 - 2xy + y^2 = x^2 + 2xy + y^2$ (47 Euc. 1) transpose and we get $x^2 = 4xy$; substitute this in the first equation and $3xy = 432\sqrt{3}$, $xy = 144\sqrt{3}$, therefore by the first $x^2 = 576\sqrt{3}$ and $x = 24\sqrt[4]{3}$, but $y = \dfrac{144\sqrt{3}}{24\sqrt[4]{3}} = 6\sqrt[4]{3}$ consequently the sides are $18\sqrt[4]{3}$, $24\sqrt[4]{3}$, and $30\sqrt[4]{3}$.

(6) *By Master* E. CHRIMES.

John and James start at the same time from the ends of the arc of a semicircular field, whose diameter is 500 yards, to walk round it. John walks at the rate of 3 miles and James at the rate of 4 miles an hour. Whereabouts in the arc will they meet, and how far from the extremities of the diameter will they be at that time?

Answered by Masters W. HEBBLETHWAITE, W. HAREWOOD, and W. PICKUP.

Here $\frac{3.1416 \times 500}{2} = \frac{1570.8}{2} = 785.4$ the length of the semicircular arc. And as 3 + 4 : 785,4 :: 3 : 336.6 yards John walked. Then 785. 4 — 336.6 = 448.8 yards James walked. Also as 3 + 4 : 180° :: 3 : 77° 8′ $\frac{4}{7}$; and 180° — 77° 8 $\frac{4}{7}$ = 102° 5′1 $\frac{3}{7}$ ∴ Sine of $\frac{77° 8 \frac{4}{7}}{2}$ or of 38° 34′ $\frac{2}{7}$ = 6234896 ∴ Sine of $\frac{102° 51\frac{3}{7}'}{2}$ or of 51° 25′ $\frac{5}{7}$ = 7818314. These Sines multiplied by the diameter viz. 500 give 311.7448 and 390.9157, their nearest distance from the extremities of the diameter.

Again by Master C. BARTHOLOMEW and C. MACANN.

As 1 : 3.1416 :: 500 : 1570.8 the circumference and half the circumference 785·4. Then according to the question as 7 : 785·4 :: 3 : 336·6, and as 7 : 785·4 :: 4 : 448.8. If the direct distances from the point of meeting to the extremities of the diameter be required, the angle formed at the centre by half the diameter and a line drawn to the point of meeting will be 77° 8′ 34″ and at the circumference 38° 34′ 17″. Then by Trigonometry as S. 90° : 500 :: S 38° 34′ 17″ : 311. 74, and S. 90° : 500 :: S. 61° 25′ 43″ : 3900·92 the distances required.

(7) *By Master D. WARDWELL.*

Two ships sail from the same port, one of them steers 100 leagues East, and the other sixty leagues North East. To find their distance from each other by Euclid 47, I only.

Answered by Masters G. ASPDEN and BAILEY SCHWARTZENBROOK.

The two ships forming an angle of 45° in their course, if a triangle be constructed one angle of which is 45 degrees, the base 100, and one of the sides 60, the perpendicular let fall on the base will be equal to that segment of the base which is a side of the angle of 45°, and the length of the hypothenuse will be 60.

Then $60 \times 60 \div 2 = 1800$; the square root of which will be 42·42. This number subtracted from the whole base leaves 57·58 and $57.58^2 + 1800 = 5115{\cdot}4564$ the square root of which being also extracted makes the answer to be 71·52.

Again by Master R. TAYLOR.

First $\frac{60^2}{2} = 1800$, and $\sqrt{1800} = 42{\cdot}42$ Then $\underline{100 - 42{\cdot}42 = 57{\cdot}58}$ therefore per Euclid 47 Book 1 $\sqrt{\overline{57{\cdot}58}^2 + \overline{42{\cdot}42}^2} = 71{\cdot}52$ the distance req. nearly.

(8) *By Master J. Locus.*

Looking at my watch I observed that it was between 9 and 10 o'clock, and that the hour and minute hands were at right angles to each other. What was the time?

Answered by Master W. VINT.

At nine o'clock the hour and minute hands are at right angles, and when the minute-hand has moved as far past six as the hour-hand is past nine they will be at right angles again. Let the space which each hand is distant, the one from 6 and the other from 9, be $= x$ Then $x + 30 = 12x$; $x = 2' \; 43'' \frac{7}{11}$. Therefore the time is $32' \; 43'' \frac{7}{11}$ past nine.

Again by Masters W. BRAMWELL, R. MOUNSEY, and G. ASPDEN.

It is manifest that the minute-hand gains 11 hours on the hour-hand in one revolution, and as they were at right angles at nine o'clock, the minute-hand must gain 30 minutes to form the given position. Therefore as $11 : 12 :: 30 : 32 \frac{8}{11}$ minutes past nine o'clock, the time required.

Number of Questions Answered by

William Hebblethwaite,	1	2	3	4	5	6	7	8	Mr. Lewthwaite's School, Halifax
Samuel Walker,	1	2	3	4	5	6	7	8	Mr. Littlewood's Acady. Roehdale
Robert Taylor,	1	2	3	4	5	6	7	8	Mr. Sherewell's School, Plymstock
William Harwood,	1	2	3	4	5	6	7	8	Melton.
William Vint,	1	2	3	4	5	6	7	8	Idle.
William Pickup,	1	2	3	4	5	6	7	8	Mr. Lightbown's Academy Union-Street, Blackburn.
Charles Macauley,	1	2	3	4	5	6	7	8	Hope School, Liverpool
John Hare,	1	2	3	4	5	6	7	8	Wetherby.
George Aspden	1	2	3	4	5	6	7	8	Blackburn.
Joel Livsey,	1	2	3	4	5	6	7	8	Mr. Armitage's Academy, Town-Meadows, Rochdale
John Grindrod,	1	2	3	4	5	6	7	8	
James Bramwell,	1	2	3	4	5	6	7	8	Mr. Huntington's Academy, Preston.
Richard Mounsey,	1	2	3	4	5	6	7	8	
Charles Bartholomew,	1	2	3	4	5	6	7	8	Woodhouse Grove, Seminary.
Christopher Macann,	1	2	3	4	5	6	7	8	Boston, Lincolnshire.
Bailey Schwartzenbrook,	1	2	3	4	5	6			Mr. Lee's Academy Windhill, near Bradford, Yorkshire.
William Brown,			3						Woodhouse Grove, Yorkshire.

ANSWERS
TO THE
MATHEMATICAL QUESTIONS

Proposed in the First Number of the Third Volume.

(183) *By* Mr. *H.* LIGHTBOWN, *Schoolmaster, Union-Street, Blackburn.*

In summing a decreasing series in geometrical progression beginning from unity, I observed, that if the three first terms were cut off, the sum of the remaining terms to infinity was equal to n times the third term. Required the series.

By Messrs. W. Godward, *Sheffield, and* Chales Holt, *Private Teacher of Mathematics, Chorley.*

Let the series be $1 + x + x^2 + x^3 +$ &c., then its sum to infinity is $\frac{1}{1-x}$, and the sum of all the terms after the 3rd. $\frac{x^2}{1-x} = nx^2$, by the question: hence $\frac{x}{1-x} = n$, or $x = \frac{n}{n+1}$, and the series is $1 + \frac{n}{n+1} + \left(\frac{n}{n+1}\right)^2 + \left(\frac{n}{n+1}\right)^3$ &c.

Corollary. The sum of the series is $= n + 1$ whatever be the value of n. For $\frac{1}{1-x} = 1 \div \left(1 - \frac{n}{n+1}\right) = n + 1$.

By Mr. F. Drake, *Colne Bridge.*

Let $1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \frac{1}{x^4}$, &c. denote the series, then its sum to infinity beginning at the third term is $\frac{1}{x^3 - x^2}$ which by the question is equal to $\frac{1}{x^2} \times n$: or $\frac{1}{x^3 - x^2} = \frac{n}{x^2}$ and $n = \frac{1}{x - 1}$ whence $x = \frac{n+1}{n}$, therefore the series is $1, \frac{n}{n+1}, \frac{n^2}{(n+1)^2}, \frac{n^3}{(n+1)^3}$ &c. as required.

By Messrs. J. Baines, and H. Lightbown, *the Proposer.*

Let the series be represented by $1, x, x^2, x^3, x^4, x^5$, &c. then, when the three first terms are cut off, the sum of the remaining terms to infinity will be expressed by $\frac{x^6}{x^3 - x^4} = \frac{x^3}{1 - x}$, hence per question, $\frac{x^3}{1 - x} = nx^2$, or $\frac{x}{1 - x} = n$; therefore $x = \frac{n}{n+1}$ and 1, $\left(\frac{n}{n+1}\right)$, $\left(\frac{n}{n+1}\right)^2$, $\left(\frac{n}{n+1}\right)^3$, $\left(\frac{n}{n+1}\right)^4$, &c. is the series required.

Note. The sum of the series is equal to $n + 1$. For, let $1 + \left(\frac{n}{n+1}\right) + \left(\frac{n}{n+1}\right)^2 + \left(\frac{n}{n+1}\right)^3 + \left(\frac{n}{n+1}\right)^4$, $+$ &c. $= S$; then by multiplying every term of the series by $\frac{n}{n+1}$, we have, $\left(\frac{n}{n+1}\right) + \left(\frac{n}{n+1}\right)^2 + \left(\frac{n}{n+1}\right)^3 + \left(\frac{n}{n+1}\right)^4 +$ &c. $= \frac{n}{n+1} \times S$; which taken from the first series, leaves, $1 = S - \frac{n}{n+1} \times S = \left(1 - \frac{n}{n+1}\right) \times S$. Consequently $S = 1 \div \left(1 - \frac{n}{n+1}\right) = n + 1$.

Ingenious solutions were also given by Messrs. J. Harker, M. Willan, S. Johnson, Wm. Stringer, J. Gillott, William Settle, James Whitehead, J. Huntington, and C. Waterhouse.

(184) *By Mr. J.* BAINES, *Nottingham Academy.*

Find the dimensions and solidity of the least parallelopipedon, whose length is x^x, breadth x^{2x}, and depth x^{3x}.

By Mr. JOHN GILLOTT, § *Sheffield.*

The solidity is x^{6x} which is to be a minimum, or x^x = a min. and in logs. $x \times$ hyp. log. x = a min. hence its fluxion $x' + x'$ hyp. log. $x = 0$, or hyp. log. $x = -1$, therefore $x = .36788$, $x^x = .6922$, $x^{2x} = .47914$, $x^{3x} = .33166$, and $x^{6x} = .11$ as required.

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§ This young man wishes to engage as Mathematical Teacher in some respectable Seminary.

By Mr. F. Drake.

The solidity will be x^{6x} which is to be a minimum, whence in fluxions $6x' +$ hyp. log $x \times 6x' = 0$, $6 +$ hyp. log. $x.6 = 0$, or hyp. log. $x = -1$, or $x = .367879$; then the dimensions will be $x^x = .6922$, $x^{2x} = .47914$, and $x^{3x} = .331662$: the solidity $x^{6x} = .11$.

In the same manner the solution is given by Messrs. J. Harker, M. Willan, S. Johnson, Wm. Stringer, Wm. Godward, Lemuel Syray, Wm. Settle, J. Whitehead, J. Huntington, C. Holt, J. Baines, and C. Waterhouse.

(185) *By Mr. W.* Godward, *Sheffield.*

Given 9 ordinates of a hyperbola between the asymptotes $\frac{8}{8}$, $\frac{8}{9}$, $\frac{8}{10}$, $\frac{8}{11}$, $\frac{8}{12}$, $\frac{8}{13}$, $\frac{8}{14}$, $\frac{8}{15}$, $\frac{8}{16}$, and the distance of the first and last ordinates 12, to find the area.

By Mr. J. Whitehead, *Huddersfield.*

By the property of the hyperbola the abscissas are inversely as their corresponding ordinates, but the least ordinate is half the greatest, therefore the greatest abscissa is double the least; that is the abscissas are 12 and 24, hence the area is $12 \times$ log. of the ratio of 12 to $24 = 12 \times$ hyp. log. $2 = 8.317766$.

By Mr. Wm. Settle, *St. James's Street, Leeds.*†

Put $\frac{8}{8} + \frac{8}{16} = a$, $\frac{8}{9} + \frac{8}{15} = b$, $\frac{8}{10} + \frac{8}{14} = c$, $\frac{8}{11} + \frac{8}{13} = d$, and $\frac{8}{12} = e$, then (*Emerson's differential method, prob.* 9. *cor.* 1.) we have

$$\frac{989a + 5888b - 928c + 10496d - 4540e}{28350} \times 12 =$$

8.31776, the area as required.

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† This gentleman has fitted up his house for the reception of a few young gentlemen as boarders.

By Messrs. J. HUNTINGTON, *Preston,* and J. GILLOTT,
Lancasterian School, Sheffield.

By page 374, Dr. Hutton's Mensuration, $A = \frac{8}{8} + \frac{8}{16} = 1 + \frac{1}{2} = 1.5$, $B = \frac{8}{9} + \frac{8}{11} + \frac{8}{13} + \frac{8}{15} = 8(\frac{1}{9} + \frac{1}{11} + \frac{1}{13} + \frac{1}{15}) = 2.764879564$. $C = \frac{8}{10} + \frac{8}{12} + \frac{8}{14} = 4(\frac{1}{5} + \frac{1}{6} + \frac{1}{7}) = 2.038095238$, and $D = \frac{12}{8} = 1.5$. Theref. $(A + 4B + 2C) \times \frac{1}{3} D = 8.31785436$ square yards nearly. Otherwise by page 380. $\frac{8}{8} + \frac{8}{16} = 1 + \frac{1}{2} = 1.5$, $\frac{8}{9} + \frac{8}{15} = 8(\frac{1}{9} + \frac{1}{15}) = 1.42222$, $\frac{8}{10} + \frac{8}{14} = 4(\frac{1}{5} + \frac{1}{7}) = 1.37142857$, $\frac{8}{11} + \frac{8}{13} = 8(\frac{1}{11} + \frac{1}{13}) = 1.34265734$, and $\frac{8}{12} \times 2 = 1.3333$. Then $A = 1.5 + 1.333 = 2.8333$, $B = 1.4222 + 1.34265734 = 2.76487956$, $C = 1.37142857 \times 2 = 2.74285714$. Then will $\frac{7A + 32B + 6C}{90} \times \frac{12}{2} = 8.31777480$ squa. yards, the area still nearer the truth.

Ingenious solutions were also given by Messrs. Wm. Stringer, Wm. Godward, C. Holt, J. Baines, C. Waterhouse, and Lemuel Syray.

(186) *By———,communicated by Mr.* E. WEBSTER.

A rivulet divides a circular field, whose diameter is 220 yards, into two parts in the ratio of 7 to 1, and is every were equidistant from a tree growing in its circumference. Find the distance of the tree from the rivulet.

By Mr. S. JOHNSON, *Apperley-bridge.*

The readiest and most certain way of solving this question is by approximation; by the rules of double position we easily find 58.415 yards for the distance of the tree from the rivulet, as required.

By Mr. J. BAINES.

Let AFC be the circular field, ABC the rivulet, and E the place of the tree; join AC, and bisect it with the perpendicular BE; then, by the question, ABC is a circular arc whose centre is E and radius EB, and the seg. ABC + seg. AEC = $\frac{1}{8}$ of the circle AFC. Suppose

EB = x, then, by the property of the circle, DE = $\frac{x^2}{220}$, and BD = $\frac{220x - x^2}{220}$; or $\frac{DE}{EF} = \frac{x^2}{220^2}$, and $\frac{BD}{2BE} = \frac{220 - x}{440}$. Assume $x = 58.4$; then area of the seg. ABC + area of the seg. AEC = $\frac{1}{8}$ of the area of the circle AFC, becomes 3568.0369 + 1181.2794 = 4751.67; hence the error is − 2.3037. Again, assume $x = 58.5$, and we have 3577.3162 + 1187.2665 = 4751.67; hence the error is + 12.9127, and by the rule of double position we obtain $x = 58.4151$ yards, the true answer.

Thus nearly were the solutions given by J. Whitehead, Wm. Godward. John Gillott, Wm. Settle, Lemuel Syray, J. Huntington, C. Holt, Charles Waterhouse, F. Drake, and E. Webster, the Proposer, who observes, "When I communicated this question to your valuable work I was not aware that it had been proposed in any other; but on looking over the Ladies' Diaries I find a very neat answer to a similar question is given by Mr. Buchanan, (See quest. 964.) Inclosed you have the solution which the gentleman gave to it who proposed it to me, with his remarks upon a similar solution to mine, and who asserts that the question can be solved without the assistance of Trial and Error. Waiting your decision, I am,*

"Sir, yours respectfully,

"Armley-Mills. "E. WEBSTER."

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* The letter containing this solution is anonymous and the writer of it unknown to us; but the solution which he asserts is done PRO FORMA is got up in such a bad form that we cannot refrain from pronouncing it erroneous.

(187) Mr. W. Teilhy, York.

A large cubical stone, standing on a horizontal plane, required a power of 5 cwt. 3 qrs. 14 lbs. when acting in the most favourable direction possible, to overturn it: Determine the direction of the power and the weight of the stone.

By the Proposer, and Mr. J. WHITEHEAD.

Let ABDC be a perpendicular section through the centre of gravity G; then it is evident from the principles of Mechanics that a given power will produce the greatest effect when applied at D in the direction DP perpen. to AD, hence the weight of the stone is

$$\frac{5\mathit{cwt.}\ 3\mathit{q.}\ 14\text{lb.} \times AD}{AK} = 5\mathit{cwt.}\ 3\mathit{q.}\ 14\text{lb.} \times 2\sqrt{2} =$$

16 *cwt.* 2*q.* 13lb.

By Messrs. J. HUNTINGTON, and C. HOLT.

Admit ABCD to be a section of the cubical stone passing through the centre of gravity G: then if the stone be supposed to revolve about A, it is obvious that the most advantageous direction DP of the sustaining power, is parallel to a tangent drawn to the arc GH, described by the point G; or it is perp. to the diagonal DGA, and conseq. makes an angle of 45° with the horizon. Theref. let W denote the weight of the stone, and P the given power $= 658$lb: then $AK : AD :: P : W$; but $AD = AB\sqrt{2}$ and $AB = 2AK$; theref. $AD = 2\sqrt{2}.\ AK$ and conseq. $AK : AK \times 2\sqrt{2}$ or $1 : 2\sqrt{2} :: P = 658$lb. $W = 1316\sqrt{2}$lb. $=$ 16 *cwt.* 2*qrs.* 13.105lb. the weight of the stone required.

Ingenious solutions were also received from Messrs. S. Johnson, Wm. Settle, J. Baines, and Francis Drake.

(188) *By Mr.* JOSHUA GARSIDE, *Mirfield.*

A ladder 9 yards long, when placed at the distance of 2 yards from the bottom of a summer-house standing at the top of a regular declivity, reached just to the top of it; but when the ladder was placed at the foot of a small insulated pillar, that stood 5 yards from the bottom of the summer-house, the top of the ladder fell two yards below the top of the building. Determine the respective heights of the summer-house and pillar without any assistance from Trigonometry, the summit of the pillar being exactly even with the bottom of the summer-house.

By Mr. F. DRAKE.

Let AD represent the summer-house, BH the pillar, EA the first position of the ladder and BC the second. Produce AD and draw BG, EF, DH perpen. to AG. Put CD $= x$, then by a known property of △s we have $BC^2\ (9^2) - BD^2\ (5^2) = CD^2 + 2CD\,.\,DG$; hence $DG = BH = \frac{56 - x^2}{2x}$; in the same manner DF is found $= \frac{73 - 4x - x^2}{2x + 4}$; but by sim. △s BD (5) : DE (2) :: DG : DF :: $\frac{56 - x^2}{2x} : \frac{73 - 4x - x^2}{2x + 4}$, that is $\frac{56 - x^2}{x} = 5 \times \frac{73 - 4x - x^2}{2x + 4}$; hence by reduction $3x^3 + 16x^2 - 253x + 224 = 0$ from which equation x is found $= 6.249424$, therefore AD $= 8.249424$ and BH $= 1.3557$.

By GEOMETRICUS.

This question considered algebraically leads to an equation of the third degree and is consequently a solid problem; but it admits of a neat and simple mechanical Construction as follows. Take the right line BD $= 5$, the given distance, also BE $= 3$, and from the centres B, E, with the radii BC, EA, each equal to 9, the given length of the ladder, describe the arcs PCL, PAK; then let a ruler be moved through the point D, till the intercepted part AC be equal to 2, the distance which the ladder fell from the top of the building: draw DH perpendicular, and BH parallel to AD: then will AD $= 8.25$ and BH $= 1.35$ yards, be the heights nearly of the summerhouse and pillar respectively.

By LEMUEL SYRAY.

Let AC $=$ PR $= 5x =$ the height of the pillar and PB $=$ height of the summerhouse, then by similar triangles DP $= 2x$, and DR $= 3x$, hence ED $= \sqrt{4 - 4x^2}$ and $\sqrt{81 - 4 + 4x^2} = \sqrt{77 + 4x^2} =$ DB; but $\sqrt{25 - 25x^2} =$ AR, and $\sqrt{81 - 25 + 25x^2} = \sqrt{56 + 25x^2} =$ SR; but since DB $+$ DR $=$ SR $+$ BS, therefore $\sqrt{77 + 4x^2} + 3x = \sqrt{25x^2 + 56} + 2$; which being cleared of surds we have $480x^3 - 3292x^2 + 3096x - 607 = 0$. This equation being solved we find $x = .27$, then $.27 \times 5 = 1.35$ equal the height of the pillar, and DB $= \sqrt{77.2916} = 8.79$: thence $8.79 - 2 \times .27 = 8.25 =$ height of the summerhouse very nearly.

Ingenious algebraical solutions were received from Messrs. J. Harker, M. Willan, S. Johnson, Wm. Stringer, Wm. Godward, J. Gillott, Wm. Settle, J. Whitehead, J. Huntington, C. Holt, J. Baines, and C. Waterhouse.

(189) *By* Mr. S. JOHNSON, *Apperley-bridge.*

How many different ways may a shilling be paid, using no other coin but two-penny pieces, penny pieces, half-pence, and farthings?

By Messrs. J. HARKER and M. WILLAN, *Leeds*, and Mr. C. HOLT, *Private Teacher, Chorley.*

This question requires us to find all the integral solutions in the indeterminate equation $8x + 4y + 2z + v = 48$.

Perhaps the readiest and most certain way of solving this equation is by substituting for x all the possible values 0, 1, 2, 3, 4, 5, 6 by which means we shall obtain the following set of equations.

$$2z + v = 48 - 4y \quad (a)$$
$$2z + v = 40 - 4y \quad (b)$$
$$2z + v = 32 - 4y \quad (c)$$

$$
\begin{aligned}
2z + v &= 24 - 4y \ (d)\\
2z + v &= 16 - 4y \ (e)\\
2z + v &= 8 - 4y \ (f)\\
2z + v &= 0 - 4y \ (g)
\end{aligned}
$$

These equations are easily adapted to the general equation $az + bv = c$, in which the number of integral solutions is equal to the greatest integer contained in the formula $\frac{a + b + c - 1}{ab}$, as is demonstrated by Mr. Barlow, at page 232, vol. 4, Leybourn's edition of the Ladies' Diaries. Thus putting $a = 2$, $b = 1$, and $c = 48 - 4y$, we have $\frac{a + b + c - 1}{ab} = \frac{1}{2}c + 1 = 25 - 2y$. Whence by giving to y all the possible values 0, 1, 2, 3, &c. we obtain the arithmetical decreasing series 25, 23, 21, 19, &c. the sum of which is $\frac{25 + 1}{2} \times 13 = 169$, the whole number of solutions that equation (a) admits of integers. Proceeding in the same manner with the remaining equations b, c, d, &c. we readily find the number of solutions which they severally admit of in integers to be equal to the squares 121, 81, 49, 25, 9, and 1. Therefore $1 + 9 + 25 + 49 + 81 + 121 + 169 = 455$, the number of ways required.

By Messrs. WHITEHEAD and C. WATERHOUSE.

Put w, x, y and z, for the number of farthings, halfpence, penny-pieces, and two-penny-pieces respectively, then we have $w + 2x + 4y + 8z = 48$ (12*d.*) or $w = 48 - 2x - 4y - 8z$. Now it is evident that z cannot be above 6; if $z = 6$, then w, x, and $y = 0$; hence in this case the question only admits of one solution: if $z = 5$, $w = 8 - 2x - 4y$, therefore if $y = 2$, $w = 0$, and $x = 0$; if $y = 1$, x may equal 0, 1, or 2; if $y = 0$, x may be 0, 1, 2, 3, or 4, that is when $x = 5$ we have $1 + 3 + 5 = 9$ different ways. In the same manner when $z = 4$, $w = 16 - 2x - 4y$, where y may be 4, 3, 2, 1 or 0: when $y = 4$, $x = 0$, when $y = 3$, x may $= 0, 1$, or 2; if $y = 2$, x may $= 0, 1, 2, 3$, or 4; if $y = 1$, x has 7 values; when $y = 0$, x has 9 values, and hence we see that the whole number of ways will be equal to 7 terms of the series $1 + (1 + 3 + 5) + (1 + 3 + 5 + 7 + 9)$ &c. $= 1 + 9 + 25 + 49 + 81 + 121 + 169 = 455$.

By Mr. W. Settle, *St. James's-street, Leeds.*

By proceeding in a manner similar to that given in the Mathematical Companion, No. 22, page 854, I have found the number of different ways 455; but although I have the whole of the operation by me, I must beg leave to omit inserting it here, the method being so very long and troublesome; but am inclined to think that some of your ingenious correspondents will favour the public with a better method of solving questions of this nature than any I have yet seen, or am at present able to suggest.

Ingenious answers were received from Messrs. S. Johnson, Wm. Godward, J. Gillotl, F. Drake, J. Baines, and Lemuel Syray.

(190) *By A. B.*

A fly runs uniformly once round the perimeter of a square whose side is 1 foot while the square makes 4 uniform revolutions about one side as an axis. How far does the fly move?

By Mr. J. Huntington, *Master of the Academy, Friargate, Preston.*

Let EBDF, represent the square, and EF the side upon which it revolves, then since the fly moves uniformly, along any one side of the square, while the square itself makes one uniform revolution round the side EF as an axis; it is evident that by reason of their compound motion the fly will describe the curvelinear line EGBIDHFE. Now in order to find its length, let $a = \text{EB} = 1$ foot, the side of the square, and $c = 6.2831853$ feet, the circum. of the end of the cylinder described by that side, then the length of EGB called Archimedes' Spiral $= \frac{1}{2} \times \overline{a^2 + c^2}|^{\frac{1}{2}} + \frac{a^2}{2c} \times$ hyp. log. of $\frac{c + \overline{a^2 + c^2}|^{\frac{1}{2}}}{a}$: that of BID, which is the spiral line of a screw; round the cylinder ABDC $= \overline{a^2 + c^2}|^{\frac{1}{2}}$: the spiral DHF = spiral EGB: and lastly, FE is a straight line $= a$. Therefore $2 \times [\frac{1}{2} \times \overline{a^2 + c^2}|^{\frac{1}{2}} + \frac{a^2}{2c} \times$ hyp. log. of $\frac{c + \overline{a^2 + c^2}|^{\frac{1}{2}}}{a}\} + \overline{a^2 + c^2}|^{\frac{1}{2}} + a = 2 \times \overline{a^2 + c^2}|^{\frac{1}{2}} + \frac{a^2}{c} \times$ hyp. log. of $\frac{c + \overline{a^2 + c^2}|^{\frac{1}{2}}}{a} + a$ which expression by restoring the value of a and c becomes $2 \times \sqrt{(1 + \overline{6.2831853}|^2)} + \frac{1}{6.2831853} \times$ hyp. log. of $[6.2831853 + \sqrt{(1 + 6.2831853)}] + 1 = 14.128382$ feet, the distance moved by the fly.

By Mr. JOHN BAINES, Jun.

It appears from a little consideration of this question, that the fly will describe the Archimedian spiral when moving along the sides of the square which are perp. to the side about which the revolution is performed; and, when moving on the opposite side, a cylindric spiral. Hence, if a = circumf. of the cylinder described by the square, and b = its radius, $\frac{1}{2} \times (a^2 + b^2)^{\frac{1}{2}} + \left(\frac{b^2}{2a} \times \text{hyp. log.} \frac{a + (a^2 + b^2)^{\frac{1}{2}}}{b}\right)$ = the length of the spiral (*Rowe's Fluxions, p.* 167, 3*rd. Ed.*), which when a

$= 2bn$ (n being $= 3.1416$). becomes $\frac{1}{2} b\,(4n^2 + 1)^{\frac{1}{2}} + \left(\frac{b}{4n} \times \text{hyp. log.}\,(2n + (4n^2 + 1)^{\frac{1}{2}}\right) = 3.38305\,b$. Also the length of the cylindric spiral is $(b^2 + (b \times 6.2832)^2)^{\frac{1}{2}} = 6.36228\ b$, and the whole space moved by the fly $(2 \times 3.38305\ b) + 6.36228\ b + b = 14.12838\,b = 14.12838$ feet. Answer.

By Messrs. SAMUEL JOHNSON *and* WM. SETTLE.

The first and third spaces passed over by the fly will evidently be the spiral of Archimedes, the second will be $\sqrt{\overline{6.2831}|^2 + 1} = 6.3622$, and the fourth will be one foot; but $\frac{1}{2} \times \overline{a^2 + b^2}|^{\frac{1}{2}} + \frac{b^2}{2a} \times$ hyperb. logar. $\overline{\frac{a + \overline{a^2 + b^2}}{b}}\Big|^{\frac{1}{2}}$ equal the length of the spiral, (*Rowe's Fluxions. page* 139,) where $a = 6.2831$ feet, and $b = 1$ foot, hence $3.1811 + .07957 \times$ hyp. log. of $12.64538 = 3.1811 + .2019 = 3.383$ feet the length of the first spiral which doubled gives $6.766 =$ the length of the first and third spirals together, cousequently $6.766 + 6.3622 + 1 = 14.1282$ feet the length of the whole space passed over by the fly; which was required.

Messrs. J. Whitehead, C. Holt, and F. Drake sent ingenious solutions to this question.

(191) *By Mr. W.* SHILLITO,

In any parabola the greatest angle contained by a tangent to the curve and the straight line joining the point of contact and the vertex. is a constant quantity. Find it.

By the Proposer, and Mr. WM. STRINGER.

Let TP, BP be the tangent and subtangent of the parabola VT. From the vertex V draw VM perpendicular to PB meeting PT in M, and on the diameter PM describe a semicircle, which because the angle PVM is a right one will pass through the point V; and since by the property of the parabola PV = BV, therefore by parallels PM = TM. Hence it is obvious that when the angle PTV is a maximum TV will be a tangent to the semicircle at V: whence drawing VO to its centre O, since the angle OVT is right, we shall have the sine of the angle OTV $= \frac{\mathrm{OV}}{\mathrm{OT}} = \frac{\mathrm{OV}}{3\mathrm{OV}} = \frac{1}{3}$: consequently the $\angle$ PTF = 19° 28′ 16″ 24‴.

The same Fluxionally by Mr. WM. GODWARD.

Put the abscissa VB $= x$, the parameter $= p$, then by the parabola the subtangent PB $= 2x$, and the ordinate BT $= \sqrt{px}$. Therefore tan. $\angle$ TPB $= \frac{\sqrt{px}}{2x}$, and tan. TVB $= \frac{\sqrt{px}}{x}$, whence tangent of their diff. or tan. $\angle$ PTV $= \left(\frac{\sqrt{px}}{x} - \frac{\sqrt{px}}{2x}\right) \div \left(1 + \frac{p}{2x}\right) = \frac{\sqrt{px}}{p + 2x}$, a maximum. This put into fluxions and reduced gives $x = \frac{1}{2}p$, therefore tan. PTV $= \frac{1}{4}\sqrt{2}$, and the $\angle$ PTV = 19° 28′ 16″ 23⅜‴ as required.

Geometrical solutions were given to this question by Messrs. Wm. Settle and J. Whitehead; and algebraical ones by Messrs. J. Gillott, S. Johnson, J. Huntington, C. Holt, J. Baines, and F. Drake.

(192) *By the same.*

A hollow cone whose vertical angle is 60°, is filled with water and placed with its base downwards. If a small orifice be made in its side so that the issuing fluid may strike the horizontal plane in a point the distance of which from the bottom of the vessel is to the distance of the orifice from the top as 5 to 4: a right line drawn from the focus of the parabola described by the spouting fluid parallel and equal to the first-mentioned distance will terminate in the cone's axis. Required the demonstration.

By Mr. JAMES WHITEHEAD.

Let EGP be the horizon, VG the side of the cone, VE its axis, O the orifice, and P the point where the water strikes the horizontal plane. Also let F be the focus of the parabola, and draw VH, CF perp. and AOD parallel to the axis VE, then per Conics VH is the directrix of the parabola OA = OF and $\angle$ FOI = AOV = 30°; but $\angle$ OIF = 120° therefore $\angle$ OFI = 30 or OI = IF. OD = $\frac{1}{2}$ OF = $\frac{1}{2}$ OA, hence OI = $\frac{1}{2}$ VO, therefore CF = CI + IF = CI + OV = $\frac{3}{4}$ VO + $\frac{1}{2}$ VO = $\frac{5}{4}$ VO, that is as 5 : 4 :: CF : VO :: (by hypothesis) GP : VO, consequently CF = GP. Q. E. D.

By Messrs. S. JOHNSON, CHARLES HOLT, *and* WM. STRINGER.

Drawing the figure as in the preceding solution let OH perpendicular to VG, a tangent to the parabola at O, meet the directrix VH in H, and join AF, FH. Then by Conics HA = HF, and AF is perpendicular to OH or parallel to VG: therefore each of the equal angles HAF, HFA is equal to the angle GVH = 60°, and consequently the triangle FAH is equilateral. But by parallels and the equilateral $\triangle$, AF = VI = AH, and FI = AV = $\frac{1}{2}$ VO, also VH = 2VO, therefore $\frac{3}{2}$ VO = AH = VI = 2CI, or CI = $\frac{3}{4}$ VO : hence CF = $\frac{3}{4}$ VO + $\frac{1}{2}$ VO = $\frac{5}{4}$ VO, or CF : VO :: 5 : 4 :: GP : VO (by hypoth.) : therefore CF = GP. Q. E. D.

By Mr. F. DRAKE.

From the algebraical process to question 137, No. 3, Vol. II, of this work, HE or the ordinate of the parabola drawn from the centre of the orifice is equal $\frac{3x}{4}$, and the distance of the said centre from the axis of the cone $= \frac{x}{2}$; consequently the distance from the axis of the cone to the axis or focus of the parabola is $\frac{5x}{4}$: also BD the distance of the spouting fluid from the base of the cone, is $= \frac{5x}{4}$, which being the same as the other distance proves the truth of the theorem. (*See the Fig. to Mr. Wiseman's solution.*)

Ingenious answers to this question were also received from Messrs J. Baines, and Wm. Settle.

(193) *By Mr. W.* Teilhy.

The sides of a triangular field are 6, 8 and 10 chains, in which are to be planted three equidistant trees, one in each fence, at the least distance from each other possible. Required their positions and distances by the principles of plane Geometry.

Answered by the Proposer.

Const. Let ABC be the given triangle right angled at A: draw AI meeting CB in I and making the angle BAI = 30°: bisect AI in G, and on BG joined demit the perpendicular IK: through K draw DV parallel to AI cutting AB in D: from D draw DE parallel to KI cutting AI in N, and CB in E; at N erect FN perpendicular to DE meeting AC in F: then will D, E, F be the positions of the trees.

Dem. Since AG = IG, by parallels KD = KV, IV = IE, and IK = DN = EN. Because the angles FAD, FND are right, the points A, D, N, F, are in a circle: therefore $\angle$ DFN = DAN = 30° (by construction): of course $\angle$ FDE = 60° = $\angle$ FED (since EN = DN and FN perp. to DE): conseq. the triangle DEF is equilateral. Now IK being a perpendicular to GB is a minimum, and therefore DE, which is double of IK, is a minimum. Q. E. D.

CALCULATION. In the tri. ACB, AB(6) : AC(8) :: radius : tangent ∠ B = 53° 7′ 48″. Then in the tri. AIB all the angles and the side AB are given to find AI = 4.83471, and BI = 3.021694. Lastly in the tri. BIG are given GI = $\frac{1}{2}$ AI = 2.41735, BI = 3.021694, and ∠ GIB = 96° 52′ 12″ to find the perp. IK = 1.77346, the double of which 3.54692 chains is the side DE required.

Thus nearly is the solution given by Geometricus, who determines the distance of the trees = 24

$$\sqrt{\frac{25 - 12\sqrt{3}}{193}} = 3.54692 \text{ chains.}$$

By Messrs. S. JOHNSON and W. SETTLE.

Let ABC be the given triangle, B the right angle, D, E, F the trees: and suppose the centres of the circles passing through the points A, F, D, and C, D, E to be at O and S respectively, from which on DF, DE, AC demit the perpendiculars OI, SG, OL, SK bisecting DF, DE, AD, DC; then LK = $\frac{1}{2}$ AC; and joining DO, DS, the right angled triangles DIO, DGS, ABC will be similar: hence OD : DI :: AC : BC, and DG : DS :: AB : AC; whence compounding these analogies, and recollecting that DI = DG, because DF = DE, by hypothesis, we have OD : DS :: AB : BC, a given ratio, and since the angle ODS is given = ODI + SDG + FDE = 90° + 60° = 150°; the tri. ODS is given in species: therefore when DI or DG is a minimum OD or DS, and consequently OS is a min. because OD : DI, DG : DS, and OS : OD are given ratios; but OS is a min. when = LK = $\frac{1}{2}$ AC, the given distance of the parallels OL, SK: hence this

CONSTRUCTION. *By Prob. III, page* 319, *Simpson's Algebra,* construct the tri. ODS having OS = $\frac{1}{2}$ AC = 5, the angle ODS = 150°, and OD : DS :: 4 : 3: then through D draw AC parallel to OS on which demit the perps. OL, SK; make AL = DL, and CK

$= DK$; and from A, C, as centres with the radii $AB = 8$, and $BC = 6$, describe arcs to intersect in B; then from O, S, to AB, CB joined, apply $OF = OD$, and $SE = SD$, so shall D, E, F, represent the places of the trees, as is evident from the Analysis.

COMPUTATION. By a known theorem in Trigonometry $4 + 3 : 4 - 3 :: \tan. \frac{\angle OSD + \angle SOD}{2} (15^\circ) : \tan. \frac{\angle OSD - \angle SOD}{2} = 2^\circ\ 11'\ 32''$, therefore $\angle OSD = 17^\circ\ 11'\ 32''$: then $S. \angle ODS = 150^\circ : OS = 5 :: S. \angle OSD : OD = 2.95575$; and per similar $\triangle$s, $AC = 10 : 2BC = 12 :: OD = 2.95575 : 2DI = DF = 3.5469$ chains, the distance required.

Solutions were also given by Messrs. J. Whitehead, J. Huntington, C. Holt. J. Baines, and F. Drake; but those gentlemen who have solved this question on the principle that two of the sides of the inscribed equilateral triangle make equal angles with the hypothenuse of the given triangle will find their solutions incorrect.

(194) *By Mr. J. H.* SWALE, *Liverpool.*

Let ABC be any plane triangle, P a given point in AC: draw AD, CE, intersecting at G, a point in BP, and meeting BC, AB in D, E; join P, D, cutting CE in F; required the Locus of L the point of concourse of AF, ED.

By the Proposer.

To render the determination of this locus extremely easy, we will demonstrate the following prismatic theorems.

If from the extremities A, C, of the side AC of any triangle ABC, there be drawn AD, CE, any two lines whatever, meeting the opposite sides of the triangle in D, E, and having their point of concourse G in a line BP given in position: the line joining the points D, E, shall pass through a given point.

Draw ED, meeting AC at M: then M is a given point. For, draw ETHO parallel to AC, cutting BP, BC, AD, in T, H, O: Because of parallels, ET : TO = CP : PA = HT : TE, and compounding, EO : ET = EH : HT, and alternately, EO : EH = ET : TH = AP : PC = AM : MC equal a given ratio, and AC a given line: that is, M is a given point.

We shew, in the same way, that, conversely, if from A a given point in CP, one of the sides of any triangle CBP, there be drawn AGD, any line whatever, meeting BP, BC, in G, D; and PD, CG, be drawn, intersecting in F; then BF tends to a given point, in the line PC.

To avoid complexity in the figure, we shall shew, that in the △ ABC, when the point M is given, and any line MDE whatever, drawn, meeting BC, BA, in D, E: then BG, drawn through G, the point of concourse of AD, CE, passes through a given point P in AC: for, from the preceding demonstration, we have universally AM : MC = AP : PC equal a given ratio, and AC given: that is, the given line AC is divided in a given ratio at P; and of course P is a given point.

Hence also, in the △ CBP, draw AGD, and PD, CG, intersecting in F: let BF meet AC at L′: Then L′ is a given point, and BL′ a line given in position.

Whence we derive the following simple enunciation: In the side CL′ of the given △ CBL′, are posited the points P, A, M; from P is drawn the line PFD, meeting BL′, BC, in F, D: and we have to determine the locus of L, the point of concourse of the lines AF, MD.

Draw LVQI parallel to CL′, cutting BC, DP, BL′, in V, Q, I, and let DI intersect CL′ at K; because of parallel lines, AP : PL′ = LQ : QI = MP : PK equal a given ratio, and MP a given line; or K a given point: that is, IV : VL = KC : CM is a given ratio.

The line IV then, is always drawn parallel to the side L′C, of the given △ L′BC, and the point L is taken in IV, making IV : IL equal a given ratio; and the locus of the point L is hence required.

Whence obviously, having determined the points M, L′, K, take L′C : CR = KC : CM, and draw BR: the required locus is the right line BR.

The locus of L will be a right line BR, under a far more general enunciation than that given in the present case. For, if BA, BP, BC, CA, be four lines given in position, and A′, C′ P′, K′, any how given in AC: draw A′GD cutting BP, BC, in G, D: also C′GE, meeting AB at E: let ED, K′F, intersect in L. The locus of L is a right line passing through B.

If AD, CE, ET, be drawn as above: then the right line passing through T, D, shall always tend to the same given point N, in AM: and which point N is determined, by making AN : NM :: NM :: NC.

In like manner, if the several lines be any how drawn, as specified in the theorem, and LI, a parallel to MA, meet BL′ at I: the line passing through the points D, I, shall always tend to the same given point K.

The position of K being such, that AP : PM :: PL′ : PK; as appears from the analysis. The line AM is divided harmonically in P and C; and AC is harmonically divided at P, L′.

This question was originally proposed in No. 13 of the Mathematical Companion, by the late R. Nicholson, of Liverpool. In the succeeding number no solution appearing, I reproposed it in Number 15, Mr. N.

having then paid the debt of nature. Owing to the usual casualties incident to Geometrical Speculation, it was still treated with unmerited neglect. It is now brought forward in the Correspondent, as a tribute of respect to the memory of that ingenious geometrician.

By Mr. WHITEHEAD,

Let the lines be drawn as per question, and the other lines as per figure; then it is already known and is easily proved that "AM is harmonically divided in P and C, but P, C are given points, therefore M is a given point" (A). In the △ ADC, GI terminates in M and in the △ PBM, HG terminates in A: hence (A), P, I, H, are in the same right line: in the same manner we prove that CLH is a right line, hence (A) AC : AM :: CK : KM, therefore K is a given point; and consequently the locus of L is the given right line BK.

COROL. If BFS the locus of F, be drawn, AM will be divided in harmonical progression by the points P, S, C, K, M. And many other curious properties of the figure might be added, did our time permit.

Ingenious solutions were received from Messrs. S. Johnson and Wm. Settle.

(195) *By Mr. R.* SMITH, *Manchester.*

At the game of Whist what are the odds of the dealer having two honours or more?

By Messrs. WHITEHEAD *and* DRAKE.

It is evident from the nature of chances that one of the three following events must happen, viz. the dealer must have 0 honour, 1 honour, or 2 or more honours. But if he has no honours all his cards must be blanks, the probability of which is $\frac{9}{13} \times \frac{47}{51} \times \frac{46}{50}$ &c. $\frac{36}{40} = \frac{19 \times 27 \times 37}{4 \times 17 \times 25 \times 49} = A$; if he has 1 honour, the card turned up must be an honour and the rest blanks, or a blank and he must receive 1 honour among the remaining 12 cards; the probability of the first event taking place is $\frac{4}{13}\times\frac{48}{51}\times\frac{47}{50}$ &c. $\frac{37}{40}=\frac{4\times19\times37}{17\times25\times49}=\frac{16A}{27}$; and of the second, is $\frac{9}{13}\times\frac{4}{51}\times\frac{47}{50}$ &c. $\frac{37}{40}\times12=\frac{9\times19\times37}{17\times25\times49}=\frac{36A}{27}$; hence adding these three probabilities together, we get $A+\frac{16A}{27}+\frac{36A}{27}=\frac{79A}{27}=\frac{19\times37\times79}{4\times17\times25\times49}=\frac{55537}{83300}$ = the probability of not having 2 or more, and $1-\frac{55537}{83300}=\frac{27763}{83300}$ the prob. of having 2 or more, therefore the odds are as 55537 to 27763 or as 2 to 1 nearly against the dealer.

By Mr. Wm. Settle, *Leeds.*

Let it be supposed that the dealer turns up an honour, the prob. of which is $\frac{4}{13}$ and (*by prob.* 20, *De Moivre's Chances,*) the prob. of the dealer having one or more honours in 51 cards will be $1-\frac{39\times38\times37}{51\times50\times49}=\frac{46744}{83300}$ therefore the prob. of having two or more honours (in the first case) will be $\frac{4}{13}\times\frac{46744}{83300}=\frac{186976}{1082900}$.

Again the prob. of not turning up an honour is $\frac{9}{13}$ and admitting this event to happen, then there remain 51 cards among which are four honours, but the prob. of the dealer having no honour by the above problem is $\frac{39\times38\times37\times36}{51\times50\times49\times48}=\frac{27417}{83300}$ and the prob. of drawing one precisely is $\frac{12\times4\times39\times38\times37}{51\times50\times49\times48}=\frac{36556}{83300}$, hence, $\frac{27417 + 36556}{83300} = \frac{63973}{83300}$ the prob. of the dealer not holding two honours, ∴ therefore $1 - \frac{63973}{83300} = \frac{19327}{83300}$, equal the prob. that he does, and $\frac{9}{13} \times \frac{19327}{83300} = \frac{173943}{1082900}$, the prob. of the dealer holding two or more honours in the second case.

Hence $\frac{186976 + 173943}{1082900} = \frac{360919}{1082900}$ the probability of the dealer holding two or more honours, therefore the odds are 721981 to 360919 against the dealer holding two or more honours, or something more than 2 to 1.

By Lemuel Syray.

First.—Suppose the turn-up card to be an honour, then we must find the probability of having one honour in taking 12 cards out of a stock of 51 cards, amongst which are 3 honours, multiplied into $\frac{4}{13}$ the probability of this supposition. Which we find by *Problem* 20 *of De Moivre's Chances.* $n = 51$, $c = 12$, $d = 39$, $p = 1$ and $a = 3$ then $\frac{12 \times 39 \times 38 \times 3}{51 \times 50 \times 49} \times \frac{4}{13} = \frac{72 \times 38 \times 2}{34 \times 1225} = \frac{5472}{41650}$.

Second.—If the card turned up be not an honour, then we must find the probability of having two honours in taking 12 cards out of a stock of 51, amongst which are 4 honours, multiplied into $\frac{9}{13}$ the probability of this case, which by the same as above $n = 51$, $c = 12$, $d = 39$, $p = 2$ and $a = 4$, then $\frac{12 \times 11 \times 39 \times 38 \times \frac{4}{1} \times \frac{3}{2}}{51 \times 50 \times 49 \times 48} = \frac{5643}{41650}$. Hence $\frac{5472 + 5643}{41650} = \frac{11115}{41650} =$ the probability of the dealer having two honours precisely. By supposing and following the same as above, we must find the probability of the dealer having 3 honours precisely, and 4 honours precisely, viz. First $n = 51$, $c = 12$, $d = 39$, $p = 2$, and $a = 3$, then $\frac{12 \times 11 \times 39 \times 3}{51 \times 50 \times 49} \times \frac{4}{13} = \frac{1584}{41650}$. Secondly, $n = 51$, $c = 12$, $d = 39$, $p = 8$, and $a = 4$ then $\frac{12 \times 11 \times 10 \times 39 \times 4}{51 \times 50 \times 49 \times 48} \times \frac{9}{13} = \frac{990}{41650}$. Hence $\frac{1584 + 990}{41650} = \frac{2574}{41650} =$ probability of the dealer having three honours precisely.

Again, $n = 51$, $c = 12$, $d = 39$, $p = 3$, and $a = 3$ then $\frac{12 \times 11 \times 10}{51 \times 50 \times 49} \times \frac{4}{13} = \frac{3520}{41650 \times 26}$. Secondly, $n = 51$, $c = 12$, $d = 39$, $p = 4$, and $a = 4$, then $\frac{12 \times 11 \times 10 \times 9}{51 \times 50 \times 49 \times 48} \times \frac{9}{13} = \frac{1485}{41650 \times 26}$. Hence $\frac{3520 + 1485}{41650 \times 26} = \frac{5005}{41650 \times 26} = \frac{385}{41650 \times 2} =$ the probability of the dealer having 4 honours precisely.

Lastly, $\frac{11115 + 2574}{41650} + \frac{385}{41650 \times 2} = \frac{27763}{83300} =$ the probability required, therefore the required odds will be as 27763 to 55537, or as 1 to 2 nearly.

It was also answered by the Proposer.

(196) *By Mr. W.* STRINGER, *Rochdale.*

Having a right line given in position parallel to the horizon and a circle given in magnitude and position in the same vertical plane; it is required to draw a tangent to the circle terminating in it and the right line, so that a heavy body may descend down it in the least time possible.

By GEOMETRICUS.

From O the centre of the given circle demit on HB the right line given in position the perpendicular OE, in which set off EI equal to the tangent EK : draw the ordinate IA perpendicular to OE, and to the point A draw the tangent AB, which will be that required.

For join BI, AO, and draw BP perpendicular to AI cutting AO in N, and OP a parallel to AI in P: then (*by Quest.* 12, *Ladies' Diary*, 1816,) BI is equal to BA, therefore the perpendicular BP will bisect the ordinate AI, and therefore by parallels it will also bisect the radius AO in N: consequently the rectangle AN.NO is a maximum; but the right angled triangles NPO, NAB being similar, the rect. $PN.NB = AN.NO$ a max. or if x be the middle of BP, then $Bx^2 - Nx^2 = PN.NB$ a max. hence since Bx $(= \frac{1}{2} BP = \frac{1}{2} OE)$ is given, it follows that Nx^2 or Nx, or BN is a minimum: consequently the time down BN, or its equal the time down BA, is a minimum.

By Mr. JOHN HUNTINGTON, *Preston.*

Let HI represent the right line given in position, CLDKC the circle given in magnitude and position, and AB the required tangent. Demit OE, and AF perp. to HI, and prolong OA to meet HI in G. Put $OE = a$, $OA = r$, $AF = x$, and $AG = y$. Then $OG = x + r$, $FG = \sqrt{(y^2 - x^2)}$, and by similar triangles AG : AF :: OG : OE; that is $y : x :: y + r : a \therefore y = \frac{rx}{a - x}$.

Also $FG : AG :: AF : AB = \frac{xy}{\sqrt{(y^2 - x^2)}} = \frac{rx}{\sqrt{[r^2 - (a-x)^2]}}$ by substitution. But the time down AB varies as $\frac{AB^2}{AF} = \frac{r^2 x}{\sqrt{[r^2 - (a - x)^2]}}$, which therefore must be a minimum: conseq. $\frac{x}{\sqrt{[r^2 - (a - x)^2]}}$ must be a minimum. In fluxions, &c. we obtain $x = \sqrt{(a^2 - r^2)}$. From whence the following construction is obvious.

Upon the perp. OE describe the semicircle OHE; and with the centre E, and rad. EK describe the arc KI. Erect IA perp. to OE, to meet the periphery of the given circle in A, through which point draw the line OG, and erect AB perp. to OG, to meet the line HI in B, and the thing is done.

Ingenious solutions were also transmitted by Messrs. Wm. Godward, Wm. Settle, Wm. Stringer, James Whitehead, C. Holt, J. Baines, C. Waterhouse, F. Drake, and Lemuel Syray.

(197) *By Mr. J. H. Swale.*

To construct the trapezium ABCD, there are given the four sides, and the angles at A and B equal to each other.

By Messrs. Samuel Johnson, *Apperley-bridge, and* W. Settle, *Teacher of Mathematics, St. James's-street, Leeds.*

Suppose the trapezium constructed, and draw CE parallel to AB meeting AD in E: then because the angles at A and B are equal, AE = BC, or ED = BC − AD, a given line. Let CD, BA meet in F; then drawing AG making the angle AGD = DEC, the triangles DEC, DGA, DAF, will be similar: hence DC : DE :: DA : DG, and DE : DC :: DA : DF; consequently DG, DF, and of course FG, FC are given lines; but the triangles FGA, FBC are similar: whence FA . FB = FG . FC; hence we have to add the line AF to the given line AB so that the rectangle FA . FB may be equal to the given rectangle FG . FC; which being done by *Prob.* 18, *Book* 5, *Simpson's Geometry*, the remaining part of the construction will be too obvious to need specifying here.

By the Proposer.

With the given sides AB, BC, CD, DA, suppose the required trapezium ABCD to be constituted, having the $\angle BAD = \angle ABC$.

Draw AE parallel to DC, also CF parallel to DA, meeting AB, AE, in F, E: and demit, upon BF, the perpendiculars CL, EP.

By hypothesis and parallels, $\angle CFB = \angle CBF$, or $CF = CB$ a given line: also ADCE is a parallelogram, having $CE = DA$, and $AE = DC$: that is, CE, AE, and of course EF, are given lines. Moreover, since $CF = CB$, and CL perpendicular to FB, we shall have $FL = LB$, and $FB = 2FL$; or $2FC : FE = 2FL : FP = FB : FP$, and consequently $PF : PB$, are given ratios.

Lastly, per a known property of triangles, $AP^2 - PF^2 = AE^2 - EF^2$ equal a given square.

Whence, AB is a given line; $BP : PF$ is a given ratio, and $AP^2 - PF^2$ is a given space.

Or the original problem reduces to the following, Having given the line AB, to add to it another line BP such, that the difference between the square of AP and a square to which BP^2 has a given ratio, shall be equal a given space.

To this problem, though an extremely easy one, we shall give the following

Comp. Perpendicular to AQ, the side of the given square, take AM, making $BA^2 : AM^2 =$ the given ratio; and to MQ apply AB: then QP, parallel to AM, determines BP.

For, per parallels, $BP^2 : PQ^2 = BA^2 : AM^2$ equal the given ratio, and AQP being a right angle, (by 47. E. I.,) $AP^2 - PQ^2 = AQ^2$ the given square by composition.

Having constructed this as an independent problem, it was unnecessary to make PQ equal to PF.

The general problem of Appollonius, on inclinations, reduces, by a short and simple analysis, to the same as the present problem.

Solutions were also given by Messrs. Wm. Stringer, W. Godward, J. Gillott, J. Whitehead, J. Huntington, C. Holt, and F. Drake.

(198) *Prize Question, by* AMICUS.

Suppose a cylindrical rod, whose length is 1000 feet and height above a horizontal plane 100 feet, to make one revolution in a horizontal position in 10 seconds of time, one end of the rod being the centre of motion; what will be the length of the curve described by a ring from the commencement of motion (which suppose is at 1 foot from the axis) till it quits the rod, and at what distance from the axis will it meet the horizontal plane? *

By the Proposer.

Let $a = 1000$, $h = 100$, $b = 10''$, $p = 3.141592$, &c. $x - 1 =$ space passed over by the ring in any variable time t, and $v =$ the corresponding velocity at that instant: then by the doctrine of central forces $4p^2b^{-2}x$ would be the velocity acquired in $1''$ by the uniform force at the distance x from the centre of motion: therefore $1'' : 4p^2b^{-2}x :: v^{-1}x' \ (t') : v'$ $\therefore$ $4p^2b^{-2}xx' = vv'$: whose correct fluent gives $v = 2b^{-1}p\sqrt{x^2 - 1}$: therefore $t' = \frac{1}{2}p^{-1}b\,(x^2 - 1)^{-\frac{1}{2}}\,x'$: moreover $b : 2px :: 1'' : 2b^{-1}px =$ velocity in the direction perpendicular to the rod: whence by compound motion $2b^{-1}p\sqrt{2} \times \sqrt{x^2 - \frac{1}{2}} = \text{V}$, the velocity in the direction of the generating curve: therefore the fluxion of the curve is $\text{V}\,t' = \sqrt{2x^2 - 1} \times (x^2 - 1)^{-\frac{1}{2}}x' = z'$; the fluent of which is the arch of the equilateral hyperbola whose semi-transverse axis $= 1$, and its abscissa $= x - 1$: hence the length of the curve might be found from the known series in Simpson's Fluxions for the Rectification of the Hyperbola; but as that series, though the best that I know of for the purpose, converges slowly when the abscissa of the curve is large, as is the case here when $x = a$, we shall investigate another series of much more rapid convergency as follows: assume $x = \sqrt{\frac{1+y}{2y}}$, then by substitution we get $z' = -\frac{1}{2} \times \frac{y^{-\frac{3}{2}}y'}{\sqrt{1-y^2}} = -\frac{y^{-\frac{3}{2}}y'}{2} - \frac{y^{\frac{1}{2}}y'}{2.2} - \frac{3y^{\frac{5}{2}}y'}{2.2.4} - \frac{3.5.y^{\frac{9}{2}}y'}{2.2.4.6} -$ &c. whence taking the fluents $z = y^{-\frac{1}{2}} - \frac{y^{\frac{3}{2}}}{2.3} - \frac{3y^{\frac{7}{2}}}{2.4.7} - \frac{3.5.y^{\frac{11}{2}}}{2.4.6.11} -$ &c. $-$ C, where C the correction of the fluent, equal the descending series $1 - \frac{1}{2.3} - \frac{3}{2.4.7} - \frac{3.5}{2.4.6.11} -$ &c., expresses the difference between the lengths of an infinite arch of the hyperbola and its asymptote, the numerical value of which is well known to be $= .5990701173$.

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* This was the Prize Question in the British Oracle; but as that excellent work was discontinued, its solution was never given to the public.

The preceding series converges so rapidly when $x = a$, that the second term $\frac{y^{\frac{3}{2}}}{2.3}$ has no value in 10 places of decimals: hence the length of the curve is $z = \sqrt{2a^2-1} - C$, extremely near $= 1414.2132088196 - .5990701173 = 1413.6141387023$, true to the last decimal place.

Now let NT represent the curve described by the ring SE a perpendicular on the tangent ET from S the centre of motion, then $ET = \frac{xx'}{z'} = \frac{xx'}{Vt} = x\sqrt{x^2-1} \div \sqrt{2x^2-1}$, which expression when $x = a$, is very nearly $= \frac{a}{2}\sqrt{2}$: therefore if T be the point where the ring quits the rod, $SE = ET = \frac{a}{2}\sqrt{2} = 707.1$.

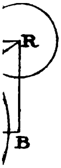
Now the velocity of the ring at T being $V = 2b^{-1} \times \sqrt{2a^2 - 1} = 888.5$ feet per second, we have by the doctrine of projectiles $V^2 \div 16\frac{1}{12} = 49084 = P$, the parameter of the parabola described by the ring; hence TP the horizontal range of the ring after it quits the rod from a point vertically under T, is by the parabola $= \sqrt{Ph} = 2215.5$; to which adding ET, gives $EP = 2922.6$: therefore $PS = \sqrt{PE^2 + SE^2} = 3007$ feet nearly, the distance required.

SCHOLIUM. Since $t' = \frac{1}{2} p^{-1} \; b(x^2 - 1)^{-\frac{1}{2}} x'$, by taking the fluent $t = \frac{1}{2} p^{-1} b \times \text{hyp. log.} (x + \sqrt{x^2 - 1})$ $=$ (when $x = a = 1000$) $= 12.0972$ seconds, the whole time from the commencement of motion till the ring quits the rod, and consequently the rod will then have made 1.20972 revolutions round S the centre of motion.

By Mr. JAMES WHITEHEAD, *Huddersfield.*

From the centre C with radius $CA = 1 = a$, describe the circular arc AB; let CBD be the rod, CQ a perp. to the tangent QDH touching the curve described by the ring at D; DH being the horizontal range of the ring after it leaves the rod, from a point vertically under D. Put $CD = x$, $DF = x'$, $DI = z$, the velocity of B in the arc $AB = b$, and $v =$ the velocity of the ring at D in the direction DL; then the centrifugal force at D will be truly expressed by $\frac{b^2 x}{a^2} = F$, but $vv' = Fx'$ (*Simpson's Fluxions, Art.* 218, *Davis' Ed.*), therefore $vv' = \frac{b^2 x x'}{a^2}$, the correct fluent of which is $v^2 = \frac{b^2}{a^2}(x^2 - a^2)$ or $v = \frac{b}{a}\sqrt{(x^2 - a^2)}$, let DL represent v, then the velocity of D in the direction DE perp. to CD being $\frac{bx}{a}$ the velocity in the tangent will be defined by DG $= \left(\frac{b^2x^2}{a^2} + \frac{b^2x^2}{a^2} - \frac{b^2a^2}{a^2}\right)^{\frac{1}{2}} = \frac{b}{a} . \sqrt{2x^2 - a^2}$. Now if $g = 16\frac{1}{12}$, we have $\frac{10}{\sqrt{g}} = t$, the time of the ring in falling to the horizontal plane, hence $\frac{b}{a}\sqrt{2x^2 - a^2} \times \frac{10}{\sqrt{g}} =$ the horizontal range $=$ DH; but DG : DE :: CD : CQ $= \frac{x^2}{\sqrt{2x^2 - a^2}}$ or neglecting a^2, CQ $= \frac{1}{2}x\sqrt{2}$ and DQ $= \frac{1}{2}x\sqrt{2}$; therefore QH $= \frac{10bx}{a\sqrt{g}}\sqrt{2} + \frac{1}{2}x\sqrt{2} = \frac{x}{a\sqrt{2g}}(a\sqrt{g} + 20b)$ and CH $= \sqrt{(\text{CQ}^2 + \text{QH}^2)} = \frac{x}{a\sqrt{2g}}\left[(a\sqrt{g} + 20b)^2 + a^2g\right]^{\frac{1}{2}}$; but $a = 1$, $b = .62832$, $g = 16\frac{1}{12}$; therefore when $x = 1000$, CH $= 3007.12$ feet. Again the fluxions of quantities being as the celerities of their increase we shall have DL $\left(\frac{b}{a}\sqrt{x^2 - a^2}\right)$: DG $\left(\frac{b}{a}\sqrt{2x^2 - a^2}\right)$ :: DF (x') : DI $(z') = x'\sqrt{\left(\frac{2x^2 - a^2}{x^2 - a^2}\right)}$, by expanding $\sqrt{\frac{2x^2 - a^2}{x^2 - a^2}}$ and taking the fluent we get $z = x\sqrt{2} \times : 1 - \frac{a^2}{4x^2} - \frac{7a^4}{96x^4} - \frac{5a^6}{128x^6} - \frac{363a^8}{14336x^8}$ &c. but when $x = a$, $z = 0$, therefore the fluent corrected is $x\sqrt{2} \times \left(1 - \frac{a^2}{4x^2} \text{ \&c.}\right) - \sqrt{2}(1 - \frac{1}{4} - \frac{7}{96} - \frac{5}{128} - \frac{363}{14336}$ &c.): which when $x = 1000$, becomes $1414.21 - \sqrt{2} \times (1 - \frac{1}{4} - \frac{7}{96}$ &c.$) = 1413.37$ the length of the curve.

Solutions on true principles were also given to the Prize Question by Messrs. F. Drake, of Colne Bridge, and Samuel Johnson, of Apperley Bridge.



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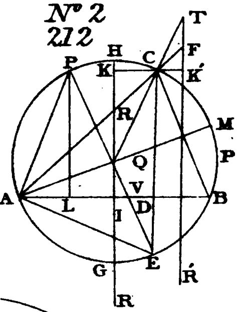

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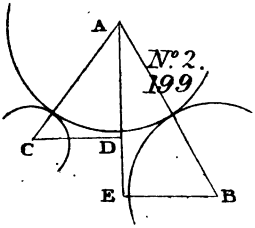

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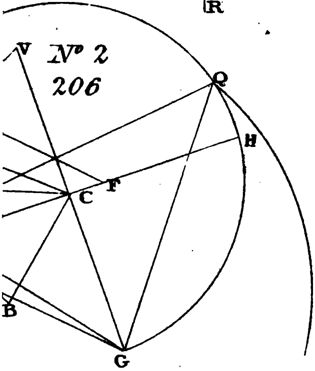

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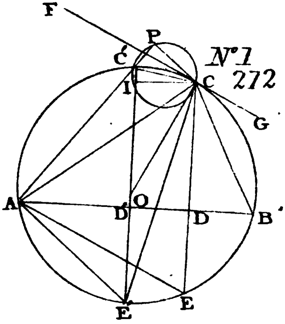

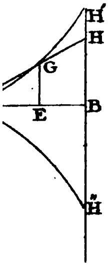

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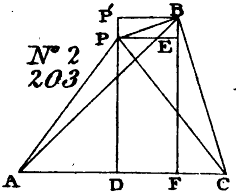

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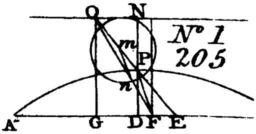

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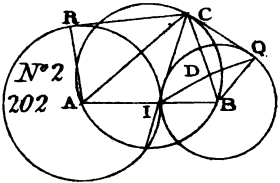

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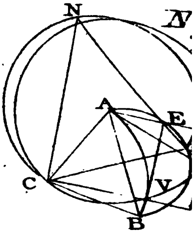

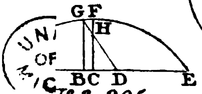

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THE

LEEDS CORRESPONDENT,

A

Literary, Mathematical, and Philosophical Miscellany.

> Tarda fluunt ingrataque tempora, quæ spem
> Consiliumque morantur agendi gnaviter
> Æque pauperibus prodest, locupletibus
> Æque neglectum pueris senibusque nocebit.
>
> HORAT.

No. III. JANUARY, 1821. Vol. III.

Translations are respectfully requested of those, among our junior correspondents, who have not attained to sixteen years of age. The extracts in each language are divided into two or three portions; that any youth, not wishing to translate the whole, may have an opportunity of applying his powers to a single portion.

JUVENILE DEPARTMENT.

Des Richesses.

On appelle richesses généralement tous les biens de la fortune; mais nous les envisagerons ici comme la possession d'une infinité de choses superflues sur un petit nombre de nécessaires. Dans ce sens, les richesses ne détournent-elles pas ceux qui les possèdent de la recherche de la vérité? N'entraînent-elles pas ordinairement la corruption des mœurs? Les passages, que nous allons citer, serviront à résoudre ces deux questions auxquelles il ne paroît pas que les philosophes aient fait jusqu'ici beaucoup d'attention quoiqu'elles les intéressent plus directement que les autres hommes.

Les richesses ont éte, pour une infinité de personnes, un grand obstacle à la philosophie. Pour jouir de la liberté d'esprit nécessaire à l'étude, il faut être pauvre, ou vivre comme les pauvres. Tout homme qui voudra mener une vie douce, tranquille et assurée, doit fuir, le plus qu'il lui sera possible, ces biens faux et trompeurs, à l'appas desquels nous nous laissons prendre comme à un trébuchet, sans pouvoir ensuite nous en détacher, en cela d'autant plus malheureux, que nous croyons les posséder, et qu'au contraire, ce sont eux qui nous possèdent et nous tyrannisent.

Depuis que les richesses ont commencé à être en honneur parmi les hommes, et à devenir, en quelque sorte, la mesure de la considération publique, le goût des choses vraiment belles et honnêtes s'est entièrement perdu. Nous sommes devenus tellement corrompus, que nous demandons, non pas de quelle utilité est une chose, mais de quel agrément. Enfin les mœurs sont si dépravées, que nous regardons la pauvreté comme un déshonneur et une infamie; en un mot, qu'elle est l'objet du mépris des riches, et de la haine des pauvres.

De continuelles inquiétudes rongent et dévorent les riches, à proportion des biens qu'ils possèdent. Tout le monde les regarde comme des gens heureux; tout le monde voudroit être à leur place: je l'avoue; mais quelle erreur! Est-il de condition pire que d'être sans cesse en bute à la misère et à l'envie? Plût aux dieux que ceux qui recherchent les richesses, interrogeassent les riches sur leur sort! Certainement ils cesseroient bientôt de désirer les richesses.

Que l'on fasse réflexion que celui qui parle dans ce passage, est un philosophe qui possédoit des biens immenses, comme il est dit dans Tacite; et l'on sentira alors de quel poids un pareil aveu doit être dans sa bouche.

Tout le mérite des hommes semble aujourd'hui être réduit aux richesses, depuis qu'elles sont devenues la passion universelle. On met le souverain bien à devenir riche, et à ne l'être que pour se satisfaire. On se regarde seul dans toute sa dépense: on considere comme perdu tout ce qui n'est point employé en meubles, en bâtimens, en plaisir. Ce n'est pas, dit Diogène, pour avoir de quoi vivre simplement avec des herbages et des fruits, qu'on cherche à s'emparer du gouvernement d'un Etat, qu'on saccage des villes, qu'on fait la guerre, aux étrangers, ou même à ses concitoyens, mais pour manger des viandes exquises, pour couvrir sa table de mets délicieux, pour avoir tous les moyens de se livrer à la débauche.

Les Scythes, dit Justin l'historien, méprisent l'or et l'argent autant que les autres hommes en sont possédés ; et c'est au mépris qu'ils font de ces vils métaux, ainsi qu'à leur manière de vivre simple et frugale, qu'il faut attribuer l'innocence et la pureté de leurs mœurs, parce que ne connoissant point les richesses, ils n'ont que faire de convoiter le bien d'autrui. Zénon le Stoïcien, disent Sénèque, Diogène, Laërce et d'autres philosophes, ne pensoit pas plus favourablement des richesses ; car ayant appris que le vaisseau, sur lequel étoient tous ses biens, avoit fait naufrage, il dit, sans témoigner aucun regret de cette perte : "La fortune veut que je "puisse philosopher plus tranquillement."

Quoique les richesses, dit encore Sénèque, soient de fort beaux présens, ce ne sont pas ceux-là que le sage désire. Il ne demande au ciel, que le simple necessaire avec la grâce de s'en servir utilement. Mais les richesses sont à présent devenues si nécessaires ; elles ont acquis un si formidable crédit ; on vient si facilement à bout de tout ce qu'on veut par elles, qu'il n'est pas surprenant que la multitude, plus exercée à pourvoir à ses besoins qu'à réfléchir sur leur source, les adore, ne croie rien au-dessus d'elles, et s'aveugle sur les moyens d'en acquérir. Le sage ne sauroit penser ainsi : il efface du nombre des heureux ceux qui ne le sont que par leurs richesses ; il n'applaudit point à l'erreur du peuple qui les croit tels.

De l'Homme Humain.

Dans nos jours passagers de peines, de misères,
Enfans du même Dieu, vivons du moins en frères
Aidons-nous l'un et l'autre à porter nos fardeaux.
Nous marchons tous courbés sous le poids de nos maux.
Mille ennemis cruels assiégent notre vie,
Toujours par nous maudite, et par nous si chérie ;
Notre cœur égaré, sans guide et sans appui,
Est brûlé de désirs, ou glacé par l'ennui.
Nul de nous n'a vécu sans connoître les larmes.
De la société les securables charmes
Consolent nos douleurs, au moins quelques instans ;
Remède encor trop foible à des maux si constans.
Ah ! n'empoisonnons pas la douceur qui nous reste !
Je crois voir des forcats dans leur cachot funeste
Se pouvant secourir, l'un sur l'autre acharnés,
Combattre avec les fers dont ils sont enchaînés.

THE POOR.

A translation from the French, of M. Bossuet.

What are the services which it is our duty to render to the poor? In what are we bound to assist them? The admirable St. Augustine gives us particular directions on this very subject:—"The service which we owe to the necessitous," says he, "is to be partakers of the burden which oppresses them. The Apostle St. Paul enjoins the faithful to bear the burdens of each other." The poor have their burden, and the rich have theirs also. The poor have their burden! who knoweth it not? When we see the sweat trickling from their foreheads, and the sound of their lamentations assails our ears, may we not acknowledge that the number of their pressing miseries constitutes a very grievous burden, with which they are over-

whelmed? But still, though the rich be walking at their ease; apparently without any thing to oppress them, be assured that they also have their burden. And what is their burden? Could you believe it?—It is their own wealth. The burden of the poor is poverty, while that of the rich is wealth. "The burden of the poor," says St. Augustine, "is to be devoid of needful sustenance; and the burden of the rich, to have more than nature requires." What then? Is it an inconvenience to be too opulent? Alas! there are worldlings, who, in the recesses of their hearts, are covetous of such a burden! But let them curb those inconsiderate desires. If the unjust prejudices of the age hinder them in this world from considering the inconveniences of abundance, when they shall enter that country where to be too rich will be a nuisance, when they shall appear at that tribunal where they will have to account not only for talents dispensed, but for those which may have been hidden, and to answer to that inexorable judge, not only for their prodigality, but for their frugality and management; then shall they know that riches are a heavy weight, then shall they regret that they neglected to distribute them.

JUVENIS.

NECESSITY THE PARENT OF THE ARTS.

A translation from the French of M. Delille.

SUCH the allotment of the thund'ring God,
Man he compelled to cultivate the sod;
Awarding only to our closest cares
The fruits which his omnipotence prepares.
He will'd that pale necessity should urge
Our talents forth, reluctant to emerge.

Till thus compell'd, around the land was seen
No bounding hedge-row or inclosure green.
Till then, unbroken did the earth remain,
One common heritage, one boundless plain.
The land without man's pains produc'd her seed,
Earth's teeming bounty far surpass'd his need.
'Twas God that would not in his wide domain
The idly rich or indolent retain.
For this the earth he harden'd, swell'd the seas,
Of fire depriv'd us, and the winds of peace;
Gave deadly venom to the viper's tongue,
The wolf against the lamb with sinews strung;
Despoil'd of honied sweets the flow'ry field,
And made the vine her purple streamlets yield.
At length, yet slowly, art reduced our pains,
And sparkling fire sprang up from flinty veins;
Now ploughshares cleave the yielding earth; the main
Is furrowed by the oar, beneath its reign.
The stars that gild heaven's bright cœrulean walls
The pilot numbers, and their titles calls.
Hounds rouse the stag; the huntsman tends his toils;
Nor can the bird escape the fowler's wiles.
The heedless fish or on the hook suspends,
Or into nets its watery circuit bends.
Soon the fire reddens in the furnace fierce;
And now my ears the file's harsh gratings pierce.
The axe despoils the branching trees, the wedge
Divides the wood with its unsharpen'd edge.
By labour all things may surmounted be;
By dint of force and keen necessity.

London. JUVENIS.

DE INDOLE.

ATTINGAM nunc quasi cunabula quædam & elementa virtutis: animique procedente tempore ad summum gloriæ cumulum perventuri, certo cum indolis experimento, datos gustus referam.

Æmilius Lepidus puer etiam tum progressus in aciem, hostem interemit, civem servavit: cujus tam memorabilis operis index est in Capitolio statua bullata, & incincta prætexta S. C. posita. Iniquum enim putavit, eum honori nondum tempestivum videri, qui jam virtuti maturus fuisset. Præcurrit igitur Lepidus ætatis stabilimentum fortiter faciendi celeritate, duplicemque laudem è prœlio retulit, cujus eum vix spectatorem anni esse patiebantur. Arma enim infesta, & destricti gladii, & discursus telorum, & adventantis equitatus fragor, & currentium exercituum impetus, juvenibus quoque aliquantum terroris incutit. Inter que Æmiliæ gentis pueritia coronam mereri, spolia rapere valuit.

Hic spiritus ne M. quidem Catonis pueritiæ defuit. Nam cum in domo M. Drusi avunculi sui educaretur, & ad eum Trib. plebis Latini de civitate impetranda convenissent; à Q. Poppedio Latinorum principe, Drusi autem hospite, rogatus, ut socios apud avunculum adjuvaret; constanti vultu non facturum se respondit. Iterum deinde, ac sæpius interpellatus, in proposito perstitit. Tunc Poppedius in excelsam ædium partem levatum, abjecturum inde se, nisi precibus obtemperaret, minatus est: nec hac re ab incepto moveri potuit. Expressa est itaque illa vox hominum, Gratulemur nobis Latini, & sooii, hunc esse tam parvum, quo senatore ne sperare quidem nobis civitatem licuisset. Tenero ergo animo Cato totius Curiæ gravitatem percepit, perseverantiaque sua Latinos jura nostræ civitatis apprehendere cupientes reppulit.

Idem cum salutandi gratia prætextatus ad Syllam venisset, & capita proscriptorum in atrium allata vidisset; atrocitate rei commotus pædagogum suum Sarpedonem nomine interrogavit, quapropter nemo inveniretur, qui tam crudelem tyrannum occideret: Cumque is, non voluntatem hominibus, sed facultatem deesse, quod salus ejus magno militum præsidio custodiretur, respondisset: ut ferrum sibi daretur, obsecravit, affirmando, perfacile se eum interfecturum, quod in lecto illius considere soleret. Pædagogus & animum Catonis agnovit, & propositum exhorruit, eumque postea ad Syllam excussum semper adduxit. Nihil hoc admirabilius. Puer in officina crudelitatis deprehensus, victorem non extimuit: tum maxime consules, Municipia, Legiones, Equestris ordinis majorem partem trucidantem. Ipsum Marium si quis eo loci statuisset, celerius aliquid de fuga sua, quam de Syllæ nece cogitasset.

EPISTOLA I.

Alius alium, ego beatissimum existimo, qui bonæ mansuræque famæ præsumptione perfruitur, certusque posteritatis cum futura gloria vivit. Ac mihi nisi præmium æternitatis ante oculos, pingue illud altumque otium, placeat. Etenim omnes homines arbitror oportere aut immortalitatem suam aut mortalitatem cogitare; et illos quidem contendere & non remitti, tum hos quiescere, nec brevem vitam caducis laboribus fatigare: ut video multos, misera simul & ingrata imagine industriæ, ad mortalitatem sui pervenire. Hæc ego tecum, quæ quotidie mecum, ut desinam mecum, si dissenties tu; quanquam non dissenties, ut qui semper preclarum aliquid et immortale meditaris. Vale.

EPISTOLA II.

Quereris de turba castrensium negotiorum, &, tanquam summo otio perfruare, lusus & ineptias nostras legis, amas, flagitas, meque ad similia condenda non mediocriter incitas. Incipio enim ex hoc genere studiorum non solum oblectationem, verum etiam gloriam petere, post judicium tuum viri gravissimi, eruditissimi, ac super ista verissimi. Nunc me rerum actus modice, sed tamen distringit: quo finito aliquid earundem Camœnarum in istum benignissimum sinum mittam. Tu passerculis & columbulis nostris inter aquilas vestras dabis pennas, si tamen & sibi & tibi placebunt: si tantum sibi, continendos cavea nidove curabis. Vale.

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A Translation from the Latin of Cicero, being the Third Part of the Fragment entitled

TIMÆUS, OR A DISCOURSE ON THE UNIVERSE.

When our juvenile readers meet with any passage which seems to countenance materialism, they must recollect that Cicero was a Heathen, and not favoured with the light of the Gospel.

By which union the heaven is so formed, as to fall under the sight and touch of mortals. For this cause therefore, and from these four things, was the body of the world formed, and constructed in that proportion which I have beforementioned; from which, it embraces itself with a certain cordial amity and affection, and so fitly adheres together that it can in no way be dissolved, but by that power by which it was at first bound together. And every part of those four things is so placed together in the whole world, that no part of that kind might project, and that all those kinds might remain in the universe. This was done for the following reasons:—in the first place, that the world, a perfect and animated being, might be composed of perfect parts; then no part being left from whence

another could be formed, that it might be one and entire; lastly, that neither any disease nor old age itself might afflict it; for the whole cementing of the body is dissolved and broken down, and reduced to diseases and old age, either by heat or cold, or by some other powerful impelling cause.

God, the Creator and Builder of the world, had therefore this reason,—that the one entire work might be completed perfect both in its whole and perfect in its parts, and thus not liable to the evils of disease or old age. But he gave it a form, particularly adapted to itself and becoming. For, wishing that all other animated beings might be contained in this animated being, he shaped it in that form in which alone all other forms can be comprised; he made it globular; what the Greeks call "sphæroides," or "after the manner of a sphere," whose every extremity is touched by the radii proceeding from its centre. And he so rounded it off, that nothing could be more rotund, in order that it might have no roughness, nothing offensive, nothing inclosed in angles, nothing rugged, no mountain, or precipice: and each part was similar to the rest, because in his judgment uniformity was more excellent than irregularity. But he surrounded the whole figure of the world with all smoothness. For it neither wanted eyes, because nothing remained on the outside that could be perceived: nor ears, because nothing could be heard: Neither were the extremities of the world encompassed with a spirit or breathing, as though it sought for respiration; nor did it require either aliment for the body, or evacuation of such food as it had consumed. For neither could any increase nor abatement be made; nor was there any thing from which either could be effected. Thus it was nourished by the consumption of itself and by its old age; since by and of itself it could endure or perform all things. For he who united these things and was their Builder, thought that the world would be content alone, and would require nothing beside itself. Therefore did he not appoint it hands, since there was nothing to be seized or to be repelled; nor feet or other members, by which it could sustain the progressive motion of the body. For he gave to the heaven that motion which was best adapted to its form, and which alone of the seven motions could restrain and keep within bounds the mind and the understanding: Therefore by one and the same revolution, it is turned round and revolves about itself. But the six remaining motions he separated from it, and by this means entirely kept it from wandering. To effect this revolution, then, which required no feet or motion, he did not endue it with the privilege of a member for walking.

This is that God who existed, and who, while thinking on another God who was at some future time to have a being, formed him smooth, and equal on every side, and similar from the centre to the surface, and perfect and complete from complete and perfect materials. But as he placed a mind in the midst of it, so he extended it throughout the whole; afterwards he surrounded it with a body, and clothed it from without: and he embraced it with a heaven which was revolved with ease and without company; and was incited to a circular motion, that it might be possible for it without difficulty to exist by itself on account of its internal power, and that, being sufficiently acquainted and familiar with itself, it might not desire another. Thus that eternal God created this God perfectly blessed. But we can scarcely assert what has just been said, that the mind had not a commencement till after the body was prepared for it; for it would not be proper for the greater to become subservient to the less.

But we say many things rashly and without consideration. God begat a mind of great antiquity, both in regard to its descent and its powers, and placed it as the lord and ruler of an obedient body, which he formed after a certain likeness. A third kind of matter was formed from two other kinds—from that which is indivisible and always of one kind and similar to itself, and from that which is divided into bodies. This kind he mingled together into one mass, both with that which was of the same nature, and that which was of a different one; and he placed it between that which was incapable of division, and that which may be divided in the body. When he had taken these three things, he tempered them into one species; and he joined with the same, that which we said was of another nature; and when, by blending with the matter that which was abhorrent and averse to the junction, of three he had formed one, he divided that one among those members to which it was most becoming.

PUER.

MARTIAL'S EPIGRAM TO LICIANUS, ON THE BIRTH-PLACE OF FAMOUS MEN.

A Translation from the Latin.

Verona loves Catullus' strains;
And Virgil glads the Mantuan swains.
Patavium of Valerius boasts,
Livy and Stella grace her coasts.
Apollodorus' strains beguile
The waters of the turbid Nile.
Ovid's sweet melody remains
Resounding from Pelignian swains.
Deserved is Cordova's praise
Of Lucan and her Senecas;
And Canius shall his fame prolong,
While Cadiz listens to his song.

Her sons to hear my Decian sing
Enchanted Merida shall bring.
My Lician too shall have renown
In Bibilis our native town;
Nor will she grudgingly refuse
The efforts of my humbler muse.

LONDON. JOHN M. HARE.

POETRY.

Dulces ante omnia Musæ. HORAT.

YOUTHFUL EXPECTATIONS.

'Tis sweet in boyhood's visionary mood,
When glowing fancy, innocently gay,
Flings forth, like motes, her bright aërial brood,
To dance and shine in Hope's prolific ray;
'Tis sweet, unweeting how the flight of years
May darkling roll in trials and in tears,
To dress the future in what garb we list,
And shape ten thousand joys that never may exist.
But he, sad wight! of all that feverish train,
Fool'd by those phantoms of the wizard brain,
Most mildly doats, when young ambition stings
To trust his weight upon poetic wings;
He downward looking in his airy ride,
Beholds Elysium bloom on every side;
Unearthly bliss each thrilling nerve attunes,
And thus the dreamer with himself communes.
'Yes! Earth shall witness, ere my star be set,
'That partial nature marked me for her pet:
'That Phœbus doomed me, kind indulgent sire!
'To mount his car, and set the world on fire.

'Fame's steep ascent by easy flights to win,
'With a neat pocket volume I'll begin;
'And dirge, and sonnet, ode and epigram,
'Shall show mankind how versatile I am.
'The buskin'd Muse shall next my pen employ:
'The boxes from their inmost rows shall sigh;
'The pit shall weep, the galleries deplore
'Such moving woes as ne'er were heard before:
'Enough—I'll leave them in their soft hysterics,
'Mount, in still brighter blaze, and dazzle with
Homerics.
'Then, while my name runs ringing through Reviews,
'And maids, wives, widows, smitten with my muse,
'Assail me with platonic *billet-doux*,
'From this suburban attic I'll dismount,
'With Coutts or Barclays open an account;
'Rang'd in my mirror, cards with bright gilt ends,
'Shall shew the whole nobility my friends;
'That happy host with whom I chuse to dine,
'Shall make set-parties, give his choicest wine;
'And age and infancy shall gape to see
'Whene'er I walk the street, and whisper, "That
'is he!"'
Poor youth! he prints—and wakes, *to sleep no more*.
The world goes on, indifferent, as before;
And the first notice of his metric skill
Comes in the likeness of—his printer's bill;
To pen soft notes no fair enthusiast stirs,
Except his laundress, and who values hers?
None but herself; for though the bard may burn
Her *note*, she still expects *one* in return.
The luckless maiden, all unblest shall sigh;
His pocket *tome* hath drawn his pocket dry.
His tragedy expires in peals of laughter;
And that soul-thrilling wish—to live hereafter,—

Gives way to one as hopeless quite, I fear,
And far more needful,—how to *live* while *here*.
Where are ye now, divine illusions all!
Cheques, dinners, wines, admirers great and small!
Chang'd to two followers, terrible to see,
Who dog him where he walks, and whisper "That
is he!" *Gent's Poems.*

EPIGRAM

To those Ladies who marry for the sake of money.

I knew a being once, his peaked head
With a few lank and greasy hairs was spread;
His visage blue, in length was like your own
Seen in the convex of a table-spoon.
His mouth, or rather gash athwart his face,
To stop at either ear had just the grace,
A hideous rift: His teeth were all canine,
And just like Death's (in Milton) was his grin.
One shilling, and one fourteen-penny leg,
(This shorter was than that, and not so big,)
He had; and they, when meeting at his knees
An angle formed of ninety-eight degrees.
Nature, in scheming how his back to vary,
A hint had taken from the dromedary:
His eyes an inward, screwing vision threw,
Striving each other thro' his nose to view.
His intellect was just one ray above
The ideot Cymon's ere he fell in love.
At school they "Taraxippus" called the wight!
The Misses, when they met him, shrieked with
fright.
But, spite of all that Nature had denied,
When sudden Fortune made the cub her pride,

And gave him twenty thousand pounds a-year,
Then from the pretty Misses you might hear,
" *His face was not the finest, and, indeed,*
He was a little, they must own, in-kneed;
His shoulders, certainly, were rather high,
But then he had a most expressive eye;
Nor were their hearts by outward charms inclined;
Give them the higher beauties of the mind."—*Ibid.*

◆

CHARACTER AND MANNERS OF THE FRENCH.

In the Museum here (Nantes) which, though small, has a very elegant collection of interesting objects, chiefly of natural history, I could not view, without feelings of horror, the dried skin of a wretched German, suspended upon the walls: he was a royalist, who behaved gallantly in support of the unfortunate Bourbons, but falling at last into the enemy's power, he was *skinned, while yet alive.* In memory of their own cruelty, the people of Nantes now display the dried and extended skin upon the walls of their Museum. There was not indeed, throughout all France, a more revolutionary or sanguinary set of people than the citizens of Nantes: to this day, a remnant of their former spirit still exists, evidently betrayed by their conversation and remarks, on deeds long since acted.

When the bills (at Abbeville) were given to each individual after dinner, one of the Frenchmen had no sooner examined the contents of that presented to him, than he flew into a violent passion with the landlord, and was so exasperated that he literally danced about the room like a madman, swearing that he would not submit to such imposition. The astonished landlord averred it was nothing more than the usual charge, when the incensed traveller threw the bill in his face, immediately upon examining it, the landlord stepped up to the Frenchman and softly whispered something in his ear, but not so softly, but that an English gentleman made out the import, which declared that the garçon in mistake had given the Frenchman a bill intended for a Monsieur Anglais. The Englishman insisted upon an explanation of the circumstance, to satisfy his suspicions, when it appeared that the bills presented to the English charged eighteen francs per head for the dinner, and those given to the French but five francs each person for the same fare.—*Stodhard's Tour.*

◆

SUPERSTITION OF THE AMERICAN INDIANS.

I was once acquainted with a white man, a shrewd and correct observer, who had lived long among the Indians, and being himself related to an Indian family, had the best opportunities of obtaining accurate information on this subject. He told me that he had found the means of getting into the confidence of one of their most noted sorcerers, who had frankly confessed to him, that his secret consisted in exciting fear and suspicion, and creating in the multitude a strong belief in his magical powers; 'For,' said he, 'such is the credulity of many, that if I only pick a little wool from my blanket, and roll it between my fingers into a small round ball, not larger than a bean, I am by that alone believed to be deeply skilled in the magic art, and it is immediately supposed that I am preparing the deadly substance with which I mean to strike some person or other, although I hardly know myself at the time what my fingers are doing; and if at that moment, I happen to cast my eyes on a particular man, or even throw a side glance at him, it is enough to make him consider himself the intended victim; he is from that instant effectually *struck*, and if he is not possessed of great fortitude, so as to be able to repel the thought, and divert his mind from it, or to persuade himself that it is nothing but the work of a disturbed imagination, he will sink under the terror thus created, and at last perish a victim, not indeed to witchcraft, but to his own credulity and folly.'

But men of such strong minds are not often to be found; so deeply rooted is the belief of the Indians in those fancied supernatural powers. It is vain to endeavour to convince them by argument that they are entirely founded in delusion and have no real existence. The attempt has been frequently made by sensible white men, but always without success. The following anecdote will shew how little hope there is of ever bringing them to a more rational way of thinking.

Sometime about the year 1776, a Quaker trader of the name of John Anderson, who among the Indians was called *the honest Quaker trader*, after vainly endeavouring to convince those people by argument that there was no such thing as witchcraft, took the bold, and I might say the rash resolution to put their sorcerers to the test, and defy the utmost exertions of their pretended supernatural powers. He desired that two of those magicians might be brought successively before him on different days, who should be at liberty to try their art on his person, and to do him all the harm that they could by magical means, in the presence of the chiefs and principal men of the village. The Indians tried at first to dissuade him from so dangerous an experiment; but he persisted, and at last they acceded to his demand; a conjurer was brought to him, who professed himself fully competent to the task for which he was called, but he could not be persuaded to make the attempt. He declared that Anderson was so good and so honest a man, so much his friend and the friend of all the Indians, that he could not think of doing him an injury. He never practised his art but on bad men and on those who had injured him: the great Mannitto forbid that he should use it for such a wicked purpose as that for which he was now called upon.

The Indians found this excuse perfectly good, and retired more convinced than ever of the abilities of their conjurer, whom they now revered for his conscientious scruples.

The one who was brought on the next day was of a different stamp. He was an arch sorcerer, whose fame was extended far and wide, and was much dreaded by the Indians, not only on account of his great powers, but of the wicked disposition of his mind. Every effort was made to dissuade Mr. Anderson from exposing himself to what was considered as certain destruction; but he stood firm to his purpose, and only stipulated that the magician should sit at the distance of about twelve feet from him, that he should not be armed with any weapon, nor carry any poison or any thing else of a known destructive nature, and that he should not even rise from his seat, nor advance towards him during the operation. All this was agreed to, the conjuror boasting that he could effect his purpose even at the distance of one hundred miles. The promised reward was brought and placed in full view, and both parties now prepared for the experiment.

" The spectators being all assembled, the sorcerer took his seat, arrayed in the most frightful manner that he could devise. Anderson stood firm and composed before him at the stipulated distance. All were silent and attentive while the wizzard began his terrible operation. He began with working with his fingers on his blanket, plucking now and then a little wool and breathing on it, then rolling it together in small rolls of the size of a bean, and went through all the antic tricks to which the power of bewitching is generally ascribed. But all this had no effect. Anderson remained cool and composed, now and then calling to his antagonist not to be sparing of his exertions. The conjuror now began to make the most horrid gesticulations, and used all the means in his power to frighten the honest Quaker, who, aware of his purpose, still remained unmoved. At last, while the eyes of all the spectators were fixed on this brave man, to observe the effects of the sorcerer's craft upon him this terrible conjuror, finding that all his efforts were vain, found himself obliged to give up the point, and alleged for his excuse 'that the Americans eat too much salt provisions; that salt had a repulsive effect, which made the powerful invisible substance that he employed recoil upon him; that the Indians, who eat but little salt, had often felt the effects of this substance, but that the great quantity of it which the white men used effectually protected them against it.

The imposition in this instance was perfectly clear and visible, and nothing was so clear as to see through this sorcerer's miserable pretence, and be convinced that his boasted art was entirely a deception; but it was not so with the Indians, who firmly believed that the salt which the Americans used was the only cause of his failure in this instance, and that if it had not been for the salted meat which Mr. Anderson fed upon, he would have fallen a victim as well as others to the incantations of this impostor.

Heckewelder's Customs of the Indians.

MAGNANIMITY REWARDED.

M. de Cortois was a very respectable clergyman, who endeavoured conscientiously to fulfil the duties of his profession. But he was a very handsome man, and in the prime of life, and was generally known in Paris by the name of "the Abbe with the handsome legs."

This agreeable outside was sufficient for the bishop of Mirepoix to form a very unfavourable opinion of him; and when he waited on him, he sought to humble him, and had firmly resolved never to give him a benefice.

The Abbé de Cortois once made a journey from Lyons to Paris in a stage-coach. He had taken the name of Quincey, which was given him by his family to distinguish him from his brothers, and by which he was known in his native place, Dijon.

There were several passengers in the coach who knew as little of him as he did of them. Most probably they had reason to be dissatisfied with M. de Mirepoix. They spoke of him with much acrimony, and painted him in very unfavourable colours. The Abbé Quincey mixed in the conversation, and defended the Bishop very zealously, and with very good arguments. He warmly praised his virtues, his good qualities, his irreproachable character, but without offending his opponents by violent contradiction, and succeeded in silencing the censurers.

Among the passengers there was an old clergyman, who had taken no part in the dispute, and did not even seem to notice it. In fact, he scarcely spoke ten words during the whole journey; but he very attentively observed the Abbé Quincey, and when they had arrived at Paris, and the travellers were getting out of the coach, the old priest familiarly pressed the Abbé's hand, and said to him; "I beg you, Abbé, to visit me in three days hence, at the convent of the Theatines, where I am a monk. Perhaps I may be so fortunate as to be of service to you, and to shew you my gratitude for having taken the part of my brother, the Bishop of Mirepoix."

The Abbé de Cortois was much surprised that he had unconsciously acquired so powerful a patron, and did not fail to call on the Abbé Boyer on the third day. The latter cordially embraced him, and said, "Go to my brother, and thank him for having applied to the king in your favour. His Majesty has appointed you Bishop de Belloy."

The Abbé de Cortois instantly waited on the Bishop of Mirepoix, who was much surprised at finding that the Abbé Quincey, who had been so strongly recommended by his brother, was no other than the Abbé de Cortois with the handsome legs, whom he had always viewed in so unfavourable a light, and so often unjustly humbled.

Living Toads found in Stones are productions of the former world.—By the Rector of Pabsdorf.

(Concluded from page 107.)

But it may be said, that these toads might have been inclosed in stone at a later period, as these animals are fond of creeping into holes and cavities of the earth in order to sleep the winter. Even the toads which were found inclosed alive in a tree must have come there in this manner. It is also known, that in limestone quarries, new rocks, as calc-tuff, &c. are formed during a comparatively short period of time, and that these animals might, perhaps, have been inclosed through these means. But if insects of a former world could be preserved in amber, and mammoths in their full flesh in ice, a toad of the primeval world could well exist alive in stone, until the present world, as it is very tenacious of life, and has the advantage of being able to pass a long time without nourishment, in a state of torpor or sleep. The fact is still a problem which naturalists or zoologists will alone be capable of solving: and which would be effected by anatomising one of these fossil toads with the view of ascertaining if it is an animal of the present or of the former world. The white colour, which the English toad had, leads us to suppose it as probable that it did not belong to our world, provided the length of time and the want of air and nourishment had not changed its natural colour and bleached its body.—In the mean time, if such an animal can exist for years in an old tree, or even in a stóne, it is also capable of being preserved in a stony prison thousands of years, because, being asleep and in so confined a situation, no exhalation takes place from it, and, therefore, there is no occasion to replace the lost animal juices by various nourishment. Wonderful phenomenon! The toad, this ugly and much despised animal, was of all others the only one capable of undergoing this experiment of nature, and, thereby, of viewing the second time the light of the world. All others, the most noble and beautiful creatures, even man himself, had it not in his power to live to see such a blessing. Man, with his fellow creatures, could only pass into the new world in a petrified state, the insects of a former world could only be preserved from complete ruin in amber, and the mammoth be partially preserved in ice, but the toad was capable, on account of its tenacious powers of life, and its peculiar nature, to pass from the old world into the new one into a living state, and by these means to be snatched from destruction. It has seen two worlds, having been an inhabitant of the old as well as the new one. It has twice trodden the theatre of the world!

How many useful considerations does the discovery not give rise to! How many weighty truths may not be traced from it!

These toads, therefore, furnish us with a fresh proof of a former world. For, if they do not belong to our world, but are different from the present animals of the same species, which, however, must be more decisively proved than at present, it is clear that there have been formerly other animals in the world than our own. Should they prove to be a new species, we shall have discovered a new race of animals of a former world, and thus add one more to those already known. It were only necessary that Cuvier should discover or examine such a toad found in stone, and perhaps one more would be added to the number of primeval animals discovered by him.

But the circumstance gives rise to other considerations; if the philospher takes pleasure in endeavouring to penetrate the depths of futurity, and in exploring the future fate of our world, and of his fellow creatures; it cannot be less agreeable and instructive to him to investigate the past, and to read the former fate of our present earthly inhabitants by the remains of a former world. Such an inquiry makes us acquainted with numerous interesting facts, and we shall now present our readers with a few of these.

We fancy ourselves standing in the subterraneous caverns of a great limestone mine, and admiring the immense masses of rock, with its different layers and strata. On nearer inspection, we find that these masses of limestone teem with millions of shell-fish, and other remains of a former world, which must have ceased to exist thousands of years ago; that we are even standing on a former bottom of the ocean, and are surrounded by millions of marine animals, and other productions of the sea. On searching, we soon find a cornu ammonis, whose species is now extinct in the world; then a nautilus, now a gryphite, or a turbinite, or a pectinite, &c. &c. In these we discover beings which have a similitude to our present inhabitants of the ocean, but are differently constructed. Here we discover a petrified fucus, and remark in it the branch of a former marine plant. There we notice the remains of an encrinite, or lily stone, and discover them to have been formerly marine animals of a remarkable nature. Here, we even find a tooth, and recognize it to have belonged to an unknown animal of the former world, or of a fish whose race has been destroyed in a great revolution of the earth. There we discover a thigh-bone lying under the ruins of the former world, and immediately pronounce it to be a part of a palaotherium. We cannot help expressing the most earnest wish to be better acquainted with this world of plants and animals for ever past away. We often, in imagination, fancy to ourselves the delight we would experience could we have seen the former world, with its various productions, in their natural and living state, in order to compare them with the present terrestrial creation! but this is a wish which cannot be gratified. We are only capable of judging, from the scanty remains of the numerous productions of that early period, of their existence and properties. If the earth is to be again inundated with water, and its inhabitants destroyed and again repeopled, the inhabitants of the new world will form nearly the same conception of the animals and vegetables of the present world as we form of those of the world which has preceded the present. But the ideas thus formed will be very imperfect. But do not let us make too hasty conclusions! On finding a piece of amber, we discover in it an insect of the former world in all its natural beauty and form, as it has lived and breathed. At another time, in breaking a rock in pieces, in order to examine its correspondent parts, and to ascertain if it contains any marine organic remains—and behold! our wish of beholding animals of the former world alive in their natural form, is now accomplished. A living creature of the former period of the earth, a toad, which has withstood the decay of thousands of years, springs out of its prison, in which it has been secured against every injury. It awakes from its slumber, on beholding the renewed light which beams around it, and of whose beneficial influence it has been so long deprived, in order to convince us of the reality of a former world, and then, after a short second existence, falls into an eternal sleep.

THE LATE HENRY ANDREWS.

This celebrated calculator was born at Frieston, near Grantham, in Lincolnshire, of poor parents. At the age of six years he would frequently stand in his shirt, looking at the moon out of the chamber window, at midnight; and when about ten years of age, he used to fix a table on Frieston Green, in clear frosty nights, and set a telescope thereon to view the stars. Soon after, he would sit for weeks together by the fire-side, with a table spread full of books, making astronomical calculations. At a suitable age he was sent from home to earn his living, and the first situation he filled was at Sleaford, as servant to a shopkeeper; after this he went to Lincoln, to wait upon a lady, and during this servitude used, at every opportunity, to make weather-glasses and weather-houses. His last situation of this kind was in the service of J. Verinum, Esq., and his master, finding him so intent upon study, allowed him two or three hours every day for that purpose. On the 1st of April, 1764, he went to Answerby Hall, the seat of Sir Christopher Witchcote, to view the great eclipse of the sun, which was visible on that day, where a number of ladies and gentlemen had assembled for the purpose; and as he had previously calculated a type of this eclipse, he presented the same to the company, shewing them the manner of its appearance in a dark room upon a board, and after it was over, they unanimously declared that his calculations came nearer the truth than any given in the Almanacks. A short time after this period he opened a school at Bassingthorpe, near Grantham, and afterwards engaged as an usher in a clergyman's boarding-school at Stilton. He then settled in Cambridge, where he proposed to reside, in the expectation that he might derive some advantage in prosecuting his studies from the men of science in the University; but the noise and bustle of the town not being agreeable to him, he left Cambridge, and came to reside at Royston, where he opened a school at the age of 23 years, and at this place continued, as schoolmaster and bookseller, until the time of his death, which happened, after a short illness, on the 26th of January, 1820, at the age of 76 years, having enjoyed an uninterrupted state of good health till his last illness. He had a very extraordinary genius for astronomy, which he cultivated through life: for more than forty years he was a computer of the *Nautical Ephemeris,* and compiler of Moore's Almanack, published by the Stationers' Company, for the same period.* He was greatly esteemed for his integrity and modesty, by every scientific man who was personally acquainted with him, or with whom he had been connected, particularly by the late Astronomer Royal (Dr. Maskelyne), who valued him much, and who, in relation to the Nautical Ephemeris, was in constant correspondence with him for nearly half a century; and also by Dr. Charles Hutton, under whose superintendence he made the Astronomical calculations of the Stationers' Almanacks.

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SINGULAR PHŒNOMENA OF EGYPT.

A strong wind that arose this day leads me to mention some particulars of the phenomena that often happen in Egypt. The first I shall notice are the *whirlwinds,* which occur all the year round, but especially at the time of the camseen wind, which begins in April, and lasts fifty days. Hence the name of camseen, which in Arabic signifies 50. It generally blows from south-west, and lasts four, five, or six days, without varying, so very strong, that it raises the sands to a great height, forming a general cloud, so thick, that it is impossible to keep the eyes open, if not under cover. It is troublesome even to the Arabs, it forces the sand into the houses through every cranny, and fills every thing with it. The caravans cannot proceed in the deserts; the boats cannot continue their voyages; and the travellers are obliged to eat sand in spite of their teeth. The whole is like a chaos. Often a quantity of sand and small stones gradually ascends to a great height, and forms a column 60 or 70 feet in diameter, and so thick, that were it steady on one spot, it would appear a solid mass. This not only revolves within its own circumference, but runs in a circular direction over a great space of ground, sometimes maintaining itself in motion for half an hour, and where it falls it accumulates a small hill of sand. God help the poor traveller who is caught under it!

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* The sale of Moore's Almanack in his hands rose to 430,000 copies per annum—yet honest Andrews never got above £25. for his labours!

The next phenomenon is the *mirage*, often described by travellers, who assert having been deceived by it, as at a distance it appears to them like water. This is certainly the fact, and I must confess that I have been deceived myself, even after I was aware of it. The perfect resemblance to water, and the strong desire for this element, made me conclude, in spite of all my caution not to be deceived, that it was really water I saw. It generally appears like a still lake, so unmoved by the wind, that every thing above is to be seen most distinctly reflected by it, which is the principal cause of the deception. If the wind agitate any of the plants that arise above the horizon of the mirage, the motion is seen perfectly, at a great distance. If the traveller stand elevated much above the mirage, the apparent water seems less united, and less deep, for, as the eyes look down upon it, there is not thickness enough in the vapour on the surface of the ground to conceal the earth from the sight. But if the traveller be on a level with the horizon of the mirage he cannot see through it, so that it appears to him clear water. By putting my head first to the ground, and then mounting a camel, the height of which from the ground might have been about ten feet at the most, I found a great difference in the appearance of the mirage. On approaching it, it becomes thinner, and appears as if agitated by the wind, like a field of ripe corn. It gradually vanishes as the traveller approaches, and at last entirely disappears when he is on the spot.

The third phenomenon is the *locusts*. These animals I have seen in such clouds, that twice the number in the same space would form an opaque mass, which would wholly intercept the rays of the sun, and cause complete darkness. They alight on fields of corn, or other vegetables, and in a few minutes devour their whole produce. The natives make a great noise to frighten them away in vain; and, by way of retaliation, they catch and eat them when fried, considering them as a dainty repast. They are something like the grasshopper in form, about two inches in length. They are generally of a yellow or gold colour; but there are some red and some green.—*Belzoni's Travels.*

DISAPPOINTED AMBITION.

A young gentleman from one of the French departments, being lately on the point of forming an advantageous union with a lady in Paris, was anxious to be distinguished in his marriage contract by the title of Chevalier. He therefore applied to a person,—a sort of universal character, styling himself a General at Genoa, a Colonel at Venice, a Duke at Rome, a Marquess at Paris, Commander of almost all the orders in the world, past, present, and future, and a member of every learned society in Europe; through the powerful influence of this gentleman, he hoped to obtain the wished for honour.

The ribbons of several foreign orders were laid before the young gentleman. He made choice of one, which, as the Marquess observed, required numerous titles, for the attainment of which many obstacles must be surmounted. "There is only one foreign Sovereign," said he, " who has the power of conferring the decoration; his secretary, however, is my most intimate friend, and probably for a douceur of about two thousand five hundred francs, it might be obtained." The young man readily paid the money, and in a short time, the General, Duke, Marquess, &c. came to congratulate him on his new dignity.

'At length, I am a knight!' exclaimed the young gentleman; 'and I have only to purchase a cross and a ribbon.' He flew to the Palais Royal, entered the shop of a jeweller, who keeps a large assortment of foreign orders, and asked for a cross of the order of the White Bear. The jeweller replied, that he had not got it. 'The Marquess informed me it was a scarce thing, thought the Knight,' as he quitted the shop. He by turns enquired at every jeweller's shop in the Palais Royal, with no better success; when he at length met with a friend, who gave him sufficient proofs of the trick by which he had been duped. He hastened to the hotel of the obliging Marquess, but alas! he had set out on the preceding evening for Constantinople.

ANSWERS
TO THE
QUESTIONS FOR YOUTH,
Proposed in the Second Number of the Third Volume.

(1) *By* Master *LAURENCE HURDMAN.*

Sold 1000 ells English for 977 guineas and gained by every yard $\frac{1}{7}$ th of what an ell Flemish cost me, what did I give for the whole?

By Master W. BIRCHELL,

Suppose the prime cost = 420, then the whole gain = 45 guineas, and therefore 465 guineas = what he sold them for, which by the question is 512 guineas, too little: Again

Suppose the cost to be 840 guineas. Then by proceeding in the same manner the error in what he sold them for is found = 47 guineas too little: Therefore

$(840 \times 512 - 420 \times 47) \div (512 - 47) = 882\frac{14}{31}$ guineas, the Answer.

By Master Hugh Huntington.

Let 1000 E.Ells $= a$, 977 guineas $= b$, and the whole cost $= x$; then $3x \div 5a =$ the cost of a F.Ell; Therefore $3x \div 35a =$ the gain per yard and $3x \div 28$ $=$ the whole gain: Consequently $x + \frac{3}{28}x = b$, and $x = \frac{28}{31}b = \frac{28}{31} \times 977 = 882\frac{14}{31}$ guineas the prime cost as required.

Remark. The 1000 Ells is a superfluous datum.

(2) *By* Master *W. Pickup.*

Given the ratio of two numbers as 2 to 3 and their difference and quotient equal to each other, to find the numbers.

By Masters W. Hebblethwaite and Ward.

Let $3x$ and $2x$ represent the required numbers.

Then by the question $3x - 2x = 3x \div 2x$. Or $x = \frac{3}{2}$. Hence the numbers are 3 and $4\frac{1}{2}$.

By Master W. Pickup.

Put $x =$ the less number, then As $2 : 3 :: x : 3x \div 2 =$ the greater number, and $3x \div 2 - x = \frac{3x}{2} \div x = \frac{3}{2}$, that is $\frac{1}{2}x = \frac{3}{2}$ and $x = 3$, the less number; then As $2 : 3 :: 3 : 4\frac{1}{2}$ the greater as required.

(3) *By* Master *W. Bramwell.*

A young gentleman from curiosity asked a shepherd the price of his choice sheep; the shepherd to avoid a direct answer, replied, x shillings and y pence; but if I am allowed to choose, you shall have one for half the aforesaid sum, which is half of y shillings together with x pence. The young man, being unskilled in the Mathematics, requires your assistance to tell him the respective values of the sheep.

By Masters John Waddington, R. Fairclough, and Thos. Boardman.

First x shillings and y pence $= 12x + y$ pence.

Second y shillings and x pence $= 12y + x$ pence.

Ergo per question $12x + y \div 2 = 6y + x$, or $12x + y = 12y + 2x$. And $x = 11y \div 10$; now since y cannot exceed 11, and must be some multiple of 10, it follows that y cannot be any other number than 10, and consequently $x = 11$; so that 11s. 10d. and 5s. 11d. are the two values of the sheep.

By Master J. Bramwell.

By the question $12x + y =$ the value of the choice sheep, and $6y + x =$ the value of the other.

Hence $12x + y = 2(6y + x) = 12y + 2x$, or $10x = 11y$, or $x = 11y \div 10$: put $y = 10$, then $x = 11$. Therefore 11s. 10d. and 5s. 11d. are the respective values.

(4) *By* Master *J.* Grindrod.

Find an arc such that the rectangle of its versed sine and cosine is a maximum.

By Master C. Bartholomew.

Let $x =$ the sine; then $1 - x =$ cosine, and $x \times 1 - x = x - x^2 =$ a maximum per question. In fluxions $x' - 2xx' = 0$, or by dividing by x' and transposing we have $2x = 1$, or $x = \frac{1}{2} =$ the cosine of 60° the arc required.

By Master W. Pickup.

Since the radius, which call 1, is to be divided into two parts, such that their rectangle may be the greatest possible, it is well known that the parts must be equal to each other; therefore the cosine of the required arc is $\frac{1}{2}$; hence the arc is 60°

(5) *By Master G. Aspden.*

Suppose A has a freehold lease of an estate from B of £100 per annum to hold 19 years and the said estate being sold for 30 years purchase to C. What must A and B each receive of the purchase money allowing 5 per cent per annum Compound Interest?

By Master Wm. Hebblethwaite.

The purchase of £100. annuity for 19 years is $12.08532 \times 100 = 1208.532$ or £1208. 10s. 7½d., and [£100. × 30yrs.] — £1208. 10s. 7½d. = £3000 — £1208. 10s. 7½d. = £1791. 9s. 4½d., A's share and £1208. 10s. 7½d. B's share.

By Master Wm. Charnley.

Let $u =$ the annuity, $R =$ the amount of £1. in a year, $t =$ the time, and P the present worth.

Then $u - (u \div R^t) \div (R - 1) = P$, the present worth, $R^t = 2.52695$, $(u \div R^t) = 100 \div 2.52695 = 39.5789$, $u - (u \div R^t) = 60.49661$ and $u - (u \div R^t) \div (R - 1) = 1208.532 = P =$ £1208. 10s. 7½d. the present worth of A's share; and $100 \times 30 =$ £3000. sold for.

Then £3000 — £1208. 10s. 7½d. = £1791. 9s. 4½d B's share.

(6) *By Master Livsey.*

Find the sum of the infinite series 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, &c.

By Master R. Mounsey and J. Waddington.

Assume $1 + \frac{1}{3}, + \frac{1}{9}, + \frac{1}{27} +$ &c. $= s$, Then by transposition $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} +$ &c $= s - 1$, and by Multiplication $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} +$ &c $= 3s - 3 = s$, According to the first assumption; Whence $3s - s = 2s = 3$, or $s = 1\frac{1}{2}$, the sum required.

By Master Wm. Charnley.

By a well-known Theorem in Geometrical progression divide the square of the first or greater term, by the difference between the said first term and the second term in the series, and the quotient is the sum of the infinite progression in finite terms.

Then $(1 \times 1) \div 1 - \frac{1}{3} = 1 \div \frac{2}{3} = \frac{3}{2} = 1\frac{1}{2}$, the sum required.

(7) *By* Master *C. Bartholomew.*

At a certain time one very fine morning last summer, I erected a stick perpendicular to a horizontal plane in the sunshine, and found the length of its shadow double its height: sometime after I inclined the stick so that it cast the longest shadow possible, and found its length also double that of the stick. Find the Sun's altitude at those times, and prove that the tangent of the less altitude was equal to the sine of the greater.

By Master H. Huntington.

Let a represent the length of the stick; then $2a =$ the length of the shadow in both cases. Therefore by Trig. and the question $2a : \text{Rad} = 1 :: a : 2 \div 2a = \frac{1}{2}$ $=$ Tan. $26^\circ\ 33'\ 54''\frac{2}{11}$, the sun's alt. in the first case, and $2a : \text{rad.} = 1 :: a : \dfrac{a}{2a} = \frac{1}{2} =$ Si. 30°, his altitude in the second case, and from the above results it is plain that the sine of the altitude in the 1st case is equal to the tangent of the Altitude in the second case. Q.E.I.

(8) *By* Master *W. Swift.*

Observed a boat sailing uniformly on the canal, which runs in a direction parallel to the window of our house through which I was looking, the figure of a man sitting upon it appeared to be in one corner of a rectangular plane, and after 3 minutes 20 seconds had elapsed he emerged out of the adjacent corner, when he blew a horn, the sound of which just reached my ear in 2 seconds. It is required to determine at what rate per hour the boat was sailing, the breadth of the pane being 6 inches, and my eye at the time of observation being posited in the perpendicular drawn to it from the first-mentioned corner, at the distance of 8 inches therefrom.

By Masters Huntington, W. Harewood, Geo. Parker, J. Gordon, Wm. Brownell, and Henry Reynolds.

It is soon found that 10 inches is the distance from the eye to the last-mentioned corner in the pane, and 1142 ft. × 2 = 2284ft, the distance of the person upon the boat when he emerged out of that corner.

Therefore 10in. : 6in, :: 2284ft. : 1370$\frac{2}{5}$ft. equal the distance sailed by the boat during the time the observations were made, whence 3′ 20″ equal 200″ : 1h. equal 3600″ :: 1370$\frac{2}{5}$: 24667$\frac{1}{5}$ft. equal 4$\frac{739}{1100}$ miles the rate per hour.

Ingenious answers were given to all the questions by Masters C. Bartholomew, Woodhouse Grove Seminary; W. Pickup, Mr. H. Lightbown's Academy, Ainsworth-street, Blackburn; W. Hebblethwaite, Mr. Lewthwaite's Academy, Halifax: W. Charnley and G. Aspden, Pupils at Pleasington School; R. Mounsey, Hugh Huntington and J. Bramwell, Trinity Church School, Preston; G. Parker, Mr. Shenstone's Academy, Stonehouse, near Plymouth; and W. Harewood, Miller, near Blackburn.

Master W. Burchall, Trinity Church School, Preston, answered 1, 2 5.

Master J Gordon, Ripon Grammar School, answered 2,8.

Master W. Brownell, Woodhouse Grove, answered 3, 8

Masters W. Ward, Rolleston Free-Grammar School, and W. Harvey, Free-Grammar School, Barnsley, answered 1, 2.

Masters H. Reynolds, W. Redsdale and Philologos, Woodhouse Grove, answered 6.

Master G. B. Mc Donald, Woodhouse Grove, answered 3.

Master, W. Sargent, Woodhouse Grove, answered 8.

ANSWERS,

TO THE

MATHEMATICAL QUESTIONS

Proposed in the Second Number of the Third Volume.

Six numbers of the Leeds Correspondent are now and will be hereafter regularly offered as a prize, to be adjudged BY LOT to one Gentleman out of those who may give true solution to the Prize Question in each number.

Mr. S. Johnson, who has answered the Prize Question in this Number, and on whom the lot fell for the Prize, will have the goodness to apply, either personally or by letter, to Mr. John Heaton, Bookseller, Briggate, Leeds, and he will receive six Copies of this Number gratuitously, as allotted to him.

(199) *By Master W. Vint, Idle.*

Two wheels turn upon another, one of which revolves twice and the other thrice in the time that the wheel on which they turn makes one revolution, and if a perpendicular of 30 inches be let fall from the centre of each exterior wheel on a line drawn through the centre of the largest wheel, the points of incidence will be 144 inches distant from each other. Required the diameter of each wheel.

By Mr. J. CROWTHER, *Frodsham, Cheshire.*

Let P, Q, R be the centres of the three circles, PA, QB equal 30 inches, the given perpendiculars on

APB. Join PQ, PR, which [Euc. 3, 12] will pass through the points of contact *m* – *n*; and draw PD perpendicular to QR. Then the radii of different circles being as their circumferences, Q*m* equal, per question, $\frac{1}{2}$P*m*. and R*m* equal $\frac{1}{3}$ P*n*[$\frac{1}{3}$P*m*]. Consequently PR equal $\frac{4}{3}$P*m*, PQ equal $\frac{3}{2}$P*m*, and P*m* equal $\frac{3}{4}$PR equal $\frac{2}{3}$PQ.

Hence, as $\frac{3}{4} : \frac{2}{3} :: 9 : 8 ::$ PQ : PR. The question is then reduced to Problem 19, Simpson's select Exercises, where we find $QD = \frac{9^2 \times QR}{9^2 - 8^2} \pm \sqrt{\left\{\left(\frac{9 \times 8 \times QR}{9^2 - 8^2}\right)^2 - DP^2\right\}}$ equal, [in the present case] 76.9736. Consequently QP equal $\sqrt{[QD^2 + DP^2]}$ equal 82.6131; and P*m* equal $\frac{2}{3}$PQ equal 55.0754. Therefore 110.1508, 55.0754 and 36.7169 inches are the three diameters required.

Iugenious solutions were also transmitted by Messrs. J. Whitehead, F. Drake, S. Ryley, C. Waterhouse. S. Johnson, W. Stringer, W. Vint, J. H. Swale, W. Settle, T. Wilkinson, J. Baines and J. Shenstone.

(200) *By Mr. J. Harker, Sheffield.*

Required the sun's azimuth, and true time of his centre appearing in the horizon in lat. 50° 50′ N. on the 26th of March, 1819.

By Mr. W. Settle, *St. James'-street, Leeds.*

It has been shewn by writers on astronomy, that the sun's centre will appear in the horizon when it is 48′ 38″ below. Then the sun's declination at that time being 1° 54′, very near, in a spheric triangle there are given the co-declination 88° 6′, the apparent zenith distance 90° 48′ 38″, and the co-latitude 39° 10′ to find the azimuth 85° 59′ 35″ from the North, and the hour angle 93° 37′ 12″; this changed into time gives 6 hours 14 minntes 29 seconds, the true time from noon when the sun's centre appeared in the horizon. Hence his apparent central rising was 5h. 45′ 31″.

But this time is 6′4″ behind the true or mean time; hence 5h. 51′ 35″ is the time required.

In the same way the solution is given by Messrs. S. Johnson, C. Waterhouse, W. Eley, S. Ryley, F. Drake, J. Baines, J. Crowther, J. Harker and J. Shenstone.

(201) *By Mr.* JOHN BAINES.

A sphere of a given radius is suspended in the air; in what latitude on the first of April, 1819, at two o'clock in the afternoon was the area of its shadow of a horizontal plane equal to its curve surface?

By the Proposer.

Put $x =$ the secant of the distance of the sun from the zenith, and r the radius of the globe; then, because the shadow is an ellipse, and the transverse diameter varies as the secant of the zenith distance, $2rx =$ the length of the shadow, $3.1416r^2x =$ its area, and $12.5664r =$ the surface of the globe.

Hence $3.1416r^2x = 12.5664r^2$, or $x = 4$ equal the natural secant of 75° 31′ 21″; therefore in an oblique angled spherical triangle are given the zenith distance equal 75° 31′ 21″, the co-declination 85° 40′, and the hour angle equal 30°, to find the co-latitude equal 11° 45′ 38″ or 158° 14′ 18″; hence the latitude is 78° 14′ 22″ N. or 68° 14′ 18″ S., either of which will answer the question.

Ingenious solutions were received from Messrs. J. Whitehead, J. Shenstone, S. Ryley, S. Johnson, F. Drake, W. Settle and J. Crowther.

(202) *By Mr. H. Lightbown.*

ACB is any plane triangle, CI the line bisecting the vertical angle: with centres A and B and radii AI, BI describe two circles, to which from C draw the tangents CQ, CR: then will CQ : CI :: CI : CR. Required the demonstration.

By Messrs. Swale, Ryley *and* Crowther.

Having described the figure agreeably to the conditions of the Theorem, let CI meet the circles at F, G; draw AQ, BR, also AK, BL perpendiculars upon CI.

Since CA : AI = CB : BI = CA : AQ = CB : BR, and the $\angle$ CQA = $\angle$ CRB, the triangles AQC, BRC are equiangular, having CA : CB = CQ : CR. Also CA : CB = IA : IB = IK : IL = FK : GL = KC : LC; hence, compounding and alternating, CF : CI = FK : GL = CA : CB = CQ : CR from above.

Whence CF . CI : CI^2 = CQ : CR = CQ^2 : CQ . CR = CQ^2 : CI^2, per the nature of tangents, that is, CQ . CR = CI^2; or obviously, CQ : CI = CI : CR.

Otherwise, by Mr. Swale.

Draw the lines QI, RI. It is well known that CA : AI = CB : BI = CA : AQ = CB : BR; also the $\angle$ CQA = $\angle$ CRB; or $\angle$ ACQ = $\angle$ BCR, and, of course, $\angle$ QCI = $\angle$ RCI.

Again, $\angle$ AIQ = $\angle$ AQI, $\angle$ BIR = $\angle$ BRI, and $\angle$ AQC + $\angle$ BRC = $\angle$ AIC + $\angle$ BIC = two right angles; then $\angle$ IQC + $\angle$ IRC = $\angle$ CIQ + $\angle$ CIR; also because of the equal angles QCI, RCI, we have $\angle$ IQC + $\angle$ QIC = $\angle$ IRC + $\angle$ RIC, that is, by adding $\angle$ IQC = $\angle$ RIC.

The angles CIQ, CRQ, are therefore equal, and consequently the $\triangle$s ICQ, ICR are equiangular, having QC : CI = CI : CR.

Mr. Swale says, the above is one of Lawson's Theorems, which, under a different enunciation, has been demonstrated in Vol. 3, Math. Repos. Old Series. We deduce from these demonstrations the following simple and elegant Porism. Let there be two circles touching each other and given in position; a third circle may be described such, that if from any point whatever in its periphery there be drawn tangents to the given circles, and also a right line to the point of contact, the line so drawn shall be a mean proportional between those tangents.

Neat demonstrations were received from Messrs. Wright, Waterhouse, Johnson, Stringer, Drake, Settle and Baines.

(203) *By Mr. T. Wooler.*

Three equal fires are placed at the angles of a plane triangle whose sides are BC = 13, AC = 14, and AB = 15 yards. How far must a person walk from the middle of the side AC in a direction perpendicular to it to feel the greatest heat?

By Mr. Baines, *Dewsbury, near Leeds.*

Draw the lines as in the figure, then from the given dimensions of the triangle we obtain EF = DG = 2, and BF = 12. Put DE = FG = x, then BG = 12 — x, $AD^2 = CD^2 = x^2 + 49$, and $BD^2 = (12-x)^2 + 4$; and because the effects of heat are inversely as the squares of the distances, we have $\frac{2}{x^2+49} + \frac{1}{(12-x)^2+4}$ a maximum. In fluxions $\frac{-4xx'}{(x^2+49)^2} + \frac{2x'(12-x)}{[(12-x)^2+4]^2} = 0$, which gives $\frac{12-x}{[(12-x)^2+4]^2}$

$= \frac{2x}{(x^2+49)^2}$, or $x^5 - 36x^4 + 614x^3 - 5128x^2 + 15403x - 9604 = 0$.

This equation has three positive roots, viz. $x = 11.998832$, 4.252, or $.8337$; the first is when the heat is a maximum, the second when it is a minimum, and the third when it is a maximum near the fires A and C, but this maximum is considerably less than the first mentioned, which is therefore the answer to the question.

⁂ Note. The converse of the well-known property, viz. that the terms of an equation, containing only real roots, are alternately + and −, is not true, for the above equation has two of its roots imaginary.

By the Proposer, Plus Minus *and* Mr. S. Ryley.

Let P be any position of the person in the right line DP bisecting AC at right angles. Then the heat from the fire at A or C being as $\frac{1}{AP^2}$ or $\frac{1}{CP^2}$, and that from the fire at B as $\frac{1}{PB^2}$, therefore the whole heat coming from all the three fires will be as $\frac{2}{PA^2} + \frac{1}{PB^2}$, a maximum: therefore $-2PB^4 . PC . P'C' - PC^4 . PB . P'B' = 0$; but $PB^2 = PE^2 + BE^2$; therefore $PB . P'B' = BE . B'E' = - BE . F'E'$; and $PC^2 = DC^2 + PD^2 (EF^2)$; therefore $PC . P'C' = EF . E'F'$. Whence by substitution and dividing by $E'F'$, we get $2PB^4 . EF - PC^4 . BE = 0$.

Hence putting $BE = x$, and finding from the data $DF = PE = 2$, we have $PB^2 = 4 + x^2$, $PC^2 = 49 + (12 - x)^2$; then by substitution and reduction there results finally $x^5 - 24x^4 + 826x^3 - 3152x^2 + 12427x - 128 = 0$.

Now the value of x in this complex equation must, from the nature of the question, be very small; for the heat at P′ (drawing BP′ perpendicular to P′D) being the greatest possible from the fire at B, will be as $\frac{1}{4}$; whilst the heat derived from both the fires at A and C will only be there as $\frac{8}{193}$, or about the $\frac{1}{24}$ part of the heat from B.

Hence it follows that the points P, P′ must nearly coalesce. Rejecting therefore the three first terms in our equation as inconsiderable, we obtain the quadratic $-3152x^2 + 12427x - 128 = 0$. Hence $x = .010327$, which subtracted from 12 gives PD $= 11.989673$, the distance required nearly.

True solutions were received from Messrs. Eley, Waterhouse, Stringer, Drake, Settle, Whitehead, and Crowther.

(204) *By Mr S. Johnson.*

Shew under what circumstances the following formulæ may be transformed into squares, viz. $m(x^2 + y^2 + z^2) - nv^2 = \square$, $m(x^2 + y^2 + v^2) - nz^2 = \square$, and $m(x + v^2 + z^2) - ny^2 = \square$.

By the Proposer.

Let $m(x^2 + y^2 + z^2) - nv^2 = a^2$, $m(x^2 + y^2 + v^2) - nz^2 = b^2$, $m(a^2 + v^2 + z^2) - n^2y^2 = c^2$, $m(v^2 + y^2 + z^2) - n^2x^2 = d^2$; from which $(m + n)x^2 + d^2 = (m + n)v^2 + a^2 = (m + n)y^2 + c^2 = (m + n)z^2 + b^2$; let $m + n = s$, and the equations become $sx^2 + d^2 = sv^2 + a^2 = sy^2 + c^2 = sz^2 + b^2$.

Now assume $c = p(fg + sde) + sq(fd - ge)$; $y = q(fg + sde) - p(fd - ge)$; $b = p(fg - sde) + sq(fd + ge)$; $z = q(fg - sde) - p(fd + ge)$; $a = p(fg + sde) - sq(fd - ge)$; $v = q(fg + sde) + p(fd - ge)$; $d = p(fg - sde) - sq(fd + ge)$; $x = q(fg - sde) + p(fd + ge)$. These assumptions will satisfy the conditions of the equations, of course there only remains this last equality, viz. $m(v^2 + y^2 + z^2) = d^2 + nx^2$.

Hence by substitution we have $d^2 + nx^2 = m \times [q^2(fg + sde)^2 - 2pq(fg + sde) \times (fd - ge) + p^2 (fd - ge)^2 + q^2(fg + sde)^2 + 2pq(fg + sde) \times (fd - ge) + p^2(fd - ge)^2 + q^2(fg - sde)^2 - 2pq (fg - sde) \times (fg + de) + p^2(fg + de)^2]$; or $2mq (fg + sde)^2 + 2mp^2(fd - ge)^2 + mq^2(fg - sde)^2 - 2mpq(fg - sde) \times (fd + ge) + mp^2(fd + ge)^2 = p^2(fg - sde)^2 - 2pq(fg - sde) \times (fd + ge) + s^2q^2(fd + ge)^2 + nq^2(fg - sde)^2 + 2npq(fg - sde) \times (fd + ge) + np^2(fd + ge)^2$; or $2mq^2(fg + sde)^2 + 2mp^2(fd - ge)^2 + (m - n) \times q^2(fg - sde)^2 - (m + n) \times 2pq(fg - sde) \times (fd + ge) + (m - n) \times p^2(fd + ge)^2 = p^2(fg - sde)^2 - 2spq(fg - sde) \times (fd + ge) + s^2q^2(fd + ge)^2$; from which, because $s = m + n$, we have $2mq^2(fg + sde)^2 + 2mp^2(fd - ge)^2 + (m - n) \times q^2(fg - sde)^2 + (m - n) \times p^2(fd + ge)^2 = p^2(fg - sde)^2 + s^2q^2(fd + ge)^2$; whence we easily obtain $q^2 = (m - n) \times (fd + ge)^2 + 2m(fd - ge)^2 - (fg - sde)^2$, and $p^2 = s^2(fd + ge)^2 - (m - n) \times (fg - sde)^2 - 2m(fg + sde)^2$.

But it appears impossible to render the proposed formulæ squares, with the assumptions we have used, when $m = n$ only in the particular case of $m = n =$ half a square, for by assuming these equal in the equation we have $p^2 \times (2mf^2d^2 + 2mg^2e^2 - f^2g^2 - 4m^3de^2) = q^2 \times (4m^2f^2d^2 + 4m^2g^2e^2 - 2mf^2g^2 - 8m^3d^2e^2)$, or $q^2(4m^2d^2 - 2mg^2) \times (f^2 - 2me^2) = p^2(2md^2 - g^2) \times (f^2 - 2me^2)$. Hence $2mq^2 = p^2$; which is evidently impossible in all cases when $2m$ is not a square.

But to return to the equations, $p^2 = s^2(fd + ge)^2 - (m - n) \times (fg - sde)^2 - 2m(fg + sde)^2$, and $q^2 = (m - n) \times (fd + ge)^2 + 2m(fd - ge)^2 - (fg - sde)^2$. Now the second side of this latter equation is easily transformed into a square if we make $(m - n) = a^2$, and $2m = b^2$; in which case $q^2 = a^2(fd + ge)^2 + b^2(fd - ge)^2 - (fg - sde)^2 = a^2(fd + ge)^2 + [b(fd - ge) + (fd - sde)] \times [b(fd - ge) - (fg - sde)]$.

Assume $q = a(fd + ge) + b(fd - ge) - (fg - sde)$; hence $q^2 = a^2(fd + ge)^2 + 2a(fd + ge) \times [b(fd - ge) - (fg - sde)] + [b(fd - ge) - (fg - sde)]^2$; now by equating this assumed square of q with the $q^2 = a^2(fd + ge)^2 + [b(fd - ge) + (fg - sde)] \times [b(fd - ge) - (fg - sde)]$ we obtain $a(fd + ge) = (fg - sde)$, from which $e = g - ad$, $f = sd + ag$; then by substitution $(fg + sde) = 2sdg - sd^2 + ag^2$, $(fg - sde = a(sd^2 + g^2)$, $(fd + ge) = sd^2 + g^2$.

Now by a careful substitution of the expressions found above into the equation of the square of p, we find $p^2 = s^2(sd^2 + g^2)^2 - a^4(sd^2 + g^2)^2 - b^2(2sdg - sad^2 + ag^2)^2 = s^2b^2d^4(b^2 - 3a^2) + 4s^2ab^2gd^3 - 2s^2b^2d^2g^2 - 4sab^2dg^3 + b^2g^4(b^2 - 3a^2)$, or $\frac{p^2}{b^2} = s^2d^4(b^2 - 3a^2) + 4s^2gd^3 - 2s^2d^2g^2 - 4sadg^3 + g^4(b^2 - 3a^2)$; which equation it is evident can always be transformed into a square when $b^2 - 3a^2$ is a square.

Now if $b^2 - 3a^2 = 1$, or $b = 2$ and $a = 1$, we have $s = 3$; and d being assumed equal 1, gives $\frac{p^2}{4} = g + 36g - 18g^2 - 12g^3 + g^4$; which is the same as the $\frac{p^2}{4}$ in question 83, No. 4, Vol 1, Leeds Correspondent.

But to proceed generally, if $b = r^2 + 3s^2$, $a = 2rs$, then $b^2 - 3a^2 = (r^2 - 3s^2)^2 = k^2$, and we have $\frac{p^2}{b^2} = s^2k^2d^4 + 4s^2agd^3 - 2s^2d^2g^2 - 4sadg^3 + k^2g^4$; assume $\frac{p}{b} = skd^2 - \frac{2sagd}{k} + kg$: hence $\frac{p^2}{b^2} =$

$s^2k^2d^4 - 4s^2agd^3 + \frac{4s^2a^2g^2d^2}{k^2} + 2sk^2d^2g^2 -$ $4sadg^3 + k^2g^4$, which must be equal to $s^2k^2d^4 + 4s^2agd^3 - 2s^2d^2g^2 - 4sadg^3 + k^2g^4$, from which the first and two last terms on each side destroying each other, and what remains being duly ordered, give $g = 4sak^2$, and $d = sk^2 + k^4 + 2sa^2$. Now it is evident the question can be varied as often as the formula $b^2 - 3a^2$ can be resolved into different squares; for $m - n = a^2$, and $2m = b^2$.

Ex. Let $b = 2$, $a = 1$, then $s = b^2 - a^2 = m + n = 3$, $k^2 = b^2 - 3a^2 = 1$, from which $m = 2$, $n = 1$; let $r = 1$, $s = 1$, then $b = 4$, $a = 2$; whence $m = 8$, and $n = 4$.

More examples might be added, but I fear I have already encroached too much upon the limited pages of your Miscellany.

Partial solutions to this very difficult problem were sent by Messrs. Ryley, Settle, Wilkinson and Baines.

(205) *By Mr. W. Shillito.*

I saw a boy throw a stone at a bird perched on the top of a tree, it just touched the bird, and fell at the distance of 60 yards from the boy, and 23 yards from the tree, which leaned directly towards the place where the boy stood, making an angle of 60° with the horizon. It is required to determine the perpendicular height of the tree, and its length, the stone being thrown in the most favourable direction possible.

By the proposer.

Const. Take AB and BE in the same right line equal to the given distances 60 and 23; at the point E make the $\angle$ AEQ = 60°; draw QN parallel to

AB at the distance $QG = \frac{1}{2}AB$, meeting EQ in Q; from which point draw QF bisecting AB in F; take any point m in QE, and from the centre m, with the radius mn equal to a fourth proportional to QE, QG and Qm, describe a circle intersecting QF in n; from F draw FP parallel to mn, meeting QE in P; then will PE be the length of the tree.

DEM. Through P draw DN perpendicular to AB. By construction the distances AB, BE and the $\angle$ AEP are of the given magnitudes: and by similar triangles $QE : QG :: QP : PN ::$ (const.) $Qm : mn :: QP : PF$; therefore $PF = PN$, and consequently the point P is in a parabola whose focus is F, parameter AB, and directrix QN. Also (by the Theory of Projectiles) the stone was thrown at the elevation of 45° which is well known to be the most favourable direction. (See Hutton's Course, Cor. 5, page 155, vol. 2, sixth edition.)

CALCULATION. In the right angled triangle QGE are given $QG = 30$, and $\angle E = 60°$, to find $QE = 34.641$, the half of which, or $17.3205 = GE$; $\therefore GF = 10.3205$; then as $QG : GF ::$ rad. : tan. $\angle GQF = 18° 59'$; this angle taken from 30° gives $\angle FQP = 11° 1'$. Now $PF : QP :: QE : QG :: S.FQP : S.\angle QFP = 12° 45'$; then $\angle FPE = 12° 45' + 11° 1' = 23° 46'$: and $S.FPE : FE :: S.PFE : PE = 17.269$. Lastly, $R. : PE :: S.\angle E : PD = 14.955$ yards, the perpendicular height of the tree.

By Mr. WM. STRINGER.

Let A be the point from which the boy threw the stone, B the point where it fell, and PG the tree. Draw AD perpendicular to AB, equal to the impetus; and join G and F, the middle of AB. Also draw DE parallel to AB, upon which demit the perpendiculars, GE and PH; and continue PG to meet DE in C; join

E, C; and produce EG to meet AB in L; and demit FI perpendicular to PC in I. Now the stone being thrown in the most favourable direction, (which I take to be when it makes an angle of 45° with the Horizon) the impetus AD is readily found to be 30 yards; also in this case it is well known that the point F is the focus of the parabolic path of the stone. In the right angled $\triangle$ CPH there are given PH $= 30$, and $\angle$ CPH $= 30°$, to find PC $= 34.642$; and in the right angled $\triangle$ FPI there are given FP $= 7$, and $\angle$ FPI $= 60°$, to find PI and FI $= 3.5$ and 6.062. Therefore IC $= 31.142$.

Again, by the property of the parabola FG $=$ GE; therefore GE (GF) : GC :: $\frac{\sqrt{3}}{2}$: 1; and in the right angled $\triangle$ CFI there are given FI $= 6.062$ and IC $= 31.142$, to find the $\angle$ FCI $= 11° \, 1'$. Therefore in the $\triangle$ FCG there are given the ratio of FG : GC, and $\angle$ FCI, to find the $\angle$ CFG $= 12° \, 45'$; and in the right angled $\triangle$ FGI there are given FI $= 6.062$ and $\angle$ FGI $= 23° \, 46'$, to find GI $= 13.769$.

Hence the length of the tree PG $=$ PI $+$ IG $= 17.969$ yards, and the perpendicular height LG $= 14.955$ yards. W.W.R.

By Messrs. Baines, Drake, Johnson *and* Waterhouse.

When the stone is thrown in the most favourable direction, the angle of elevation is 45°, and the range equal to 4 times the height. Hence AC $=$ CE $= 30$, CF $= 15$, and CD $= 7$. Put FH $= x$, then CF : AC² :: FH : GH² $=$ BC² $= 60x$; therefore BC $= \sqrt{60x}$, BD $= 7 + \sqrt{60x}$, and CH $=$ BG $= 15 - x$; but by Trig. sine $\angle$ BGD : BD :: sine $\angle$ BDG : BG $= 7\sqrt{3} + \sqrt{180x}$; hence $7\sqrt{3} + \sqrt{180x} = 15 - x$, or $x = 150 - 7\sqrt{3} - 6\sqrt{(300 - 35\sqrt{3})} = .04455$

$15 - x = 14.95545$ yards, the perpendicular height of the tree, and 17.2691 yards, the length.

Ingenious Algebraical solutions were transmitted to this question by Messrs. Ryley, Settle, Whitehead, Crowther and Shenstone.

(206) *By Mr. J. H.* Swale, *Liverpool.*

Given the base line from the vertical angle bisecting the base, and line bisecting the vertical angle and terminating in the base, to construct the plane triangle.

(This question has been proposed before; but I think it merits a better solution than it has yet received.)

By Plus Minus.

Suppose it done, and that ACB, the required triangle is circumscribed by a circle of which the diameter EF bisects the given base AB in M, and let the given bisecting lines CM, CD be produced to meet the circle again in G,E; join EG, FC, and produce EG, AB to meet in P. Since the angles EMD and ECF are right ones, and the angle E is common to the two triangles EMD, ECF, they are therefore equiangular.

Hence $\angle$ EDM = EFC = (Euc. 3, 21) $\angle$ EGC: therefore $\angle$ PGC = $\angle$ PDC, and consequently the circumference of a circle might pass through the points P, G, D, C; therefore the rectangle PM . MD = GM . MC = AM^2 equal a given square, and because MC is given, GM is given; but in consequence of the similar triangles PMG, CMD, the ratio of PM to PG will be the same as that of CM to CD, which ratio is therefore given, and the points G, M being given, the locus of the point P will be a given circle. Also since the locus of the point D is a given circle whose centre C is a given point in the straight line CG, given in position and length, our problem is now reduced to another, which is well known, and may be thus enunciated:

To draw through the given point M, a straight line PD intersecting two given circles in the points P, D, and making the rectangle PM . MD equal to $(AM)^2$ a given square. Hence this

Construction. Take CM equal to the given line bisecting the base, in which produced find GM a third proportional to CM and AM half the given base; then (by Prob. 13, Simpson's Geometry) describe the semicircle QPR, so that any two lines being drawn from the points G, M to meet in its periphery, shall obtain the ratio of the given bisecting lines. And (by prob. 25, ibid.) let PD be drawn through M, cutting the semicircle QPR in P, and the circle described to centre C and radius CD in D, so that the rectangle PMD shall be equal to the square of half the given base. In this line take MA and MB each equal to half the given base; then joining AC, BC, the thing required will be done, as is evident from the Analysis.

By Mr. SWALE.

Comp. Take CH equal CL, and CE equal the respective bisecting lines, also perpendicular CG equal perpendicular EP equal half the given base, and join G, E. From the circle passing through the points G, H, L cut the segment GHQ to contain an ∠ equal GEC; then the circle through Q, P to contain an ∠ equal EGC, determines R, one extremity of the diameter of the circumscribing circle bisecting the base.

To a semi-circle passing through the points R, E apply CD equal CH or CL; in DE take DA equal DB equal CG or EP; join AC BC; we have to shew that ACB is the △ required.

Draw RP, RD, RQ; from M, the intersection of RQ, with the circle GHL, draw MIG; join I, D; produce RD to CV, a perpendicular to CR, at V; and draw PF, a perpendicular to PR, to RH at F; RH, MG intersecting at I.

By the Comp. AB is equal the given base, and CD, CE equal the respective bisecting lines: we only have to shew that the $\angle$ ACE equal $\angle$ BCE.

By the Comp. $\angle$ GMQ equal $\angle$ GEC, or $\angle$ GMR equal $\angle$ GER, and therefore the points G, E, M, R; as also G, H, M, L are in a circle.

That is, $GI . IM = EI . IR = HI . IL = ID^2$, because the points L, D, H, as also E, D, R are in a semi-circle.

The line ID is therefore perpendicular to RE, and, of course, parallel to CV: then $ER : RD = DV : IC$, or $RD . DV = ER . IC$.

Again, as $\angle$ PRQ equal $\angle$ EGC, and $\angle$ QRF equal $\angle$ IGE, we shall have the angle PRF equal $\angle$ CGI equal $\angle$ EPF, or GI parallel to PF: and the $\triangle$s EPF, CGI mutually equal, having $EF = CI$.

Hence $ER . IC = ER . EF = EP^2 = DA^2 = AD . DB = RD . DV$ from above; and, of course, the points R, A, V, C, B are in a circle to the diameter RV.

Whence, obviously, as $AD = DB$ and VR perpendioular to AB, the $\angle ACR = \angle BCR$, or $\angle ACE = \angle BCE$.

By Mr. W. Wright *of Hull.*

Analysis. Suppose ACB the triangle sought, GH bisecting AB at D, the diameter of its circumscribing circle; draw CD, CH and CG cutting AB at E, and bisecting the $\angle$ ACB; also draw DF perpendicular to CG. By similar triangles $DG : EG :: CF : DH$, or $EG \times CF = DG \times DH = BD^2$ by the circle; also by right angled triangles $DF^2 = EF \times FG$, and $CD^2 = CF^2 + DF^2 = CF^2 + EF \times FG$, but $CF = CE + EF$; ergo $(CE + EF)^2 + EF \times FG = CE^2 \times 2CE \times EF + EF^2 + EF \times FG = CD^2$.

Again, $EG = EF + FG$; ergo $EG \times CF = (EF + FG) \times (CE + EF) = EF \times CE + EF^2 + CE \times FG + EF \times FG = BD^2$, taking equal quantities from equal quantities leaves $CE + CE \times EF - CE \times FG = CD^2 - BD^2$, or $CE \times (FG - EF) = CE^2 + BD^2 - CD^2 =$ a given space, and CE being given, $FG - EF$ is determinable by lineal sections; in FG make $IF = EF$, then $GI = FG - EF$, and $EG = 2EF + IG$, and we have shewn above that $EG \times CF = BD^2$, viz. $(2EF + IG) \times (CE + EF) = 2CE \times EF + 2EF^2 + CE \times IG + EF \times IG = BD^2$, or $EF^2 + EF \times (CE + \frac{1}{2}IG) = BD^2 - CE \times IG$; here CE, IG, BD are known, and we may now derive the following

CONSTRUCTION. Find IG such that $IG \times CE = CE^2 + BD^2 - CD^2$, likewise determine EF so that $EF^2 + EF \times (CE + \frac{1}{2}IG) = BD^2 - CE \times IG$; in GI produced lay off $IE = 2EF$; on GE as a diameter describe a semi-circle cutting at D, a perpendicular to GE at F; from D to GE prolonged apply $DC =$ the line bisecting the base; through D and E draw each way $DA = DB =$ half the given base; join C with A and B, then is ACB the △ sought.

Answers were received from Messrs. Johnson, Settle, Waterhouse and Baines.

(207) *By* DIOPHANTUS.

Find the sides of a right angled plane triangle such that its perimeter shall be a square, and if to the square of each of its sides the sum of the other two be added, the numbers thence arising shall be all squares.

By Mr. J. WHITEHEAD *and* Mr. THOMAS WILKINSON.

Let w, x, y be the three sides of the triangle, and put $w + x + y = n$; then $w^2 + x + y = w^2 - w +$

$n = \square$, $y^2 + w + x = y^2 - y + n = \square$, $x^2 + w + y = x^2 - x + n = \square$.

Now these three formulæ are evidently all squares when $n = \frac{1}{4}$; assume therefore $w + x + y = \frac{1}{4}$. and we have only to make $w^2 + x^2 = y^2$, for this purpose put w equal $2amn$, x equal $a\,(m^2 - n^2)$, y equal $a\,(m^2 + n^2)$, and we have $w + x + y = 2am(m + n) = \frac{1}{4}$, or $a = \dfrac{1}{8m\,(m+n)}$, hence $w = \dfrac{2mn}{8m\,(m+n)}$, $y = \dfrac{m^2 + n^2}{8m\,(m+n)}$, and $x = \dfrac{m^2 - n^2}{8m\,(m+n)}$; let $m = 2$, $n = 1$, then $w = \frac{4}{48}$, $x = \frac{3}{48}$ and $y = \frac{5}{48}$.

Thus the question is answered by Messrs. Ryley, Wright, Johnson, Drake, Settle, and Baines.

(208) *By* DIOPHANTUS.

It is required to find 4 square numbers having these *fourteen* remarkable properties, viz. such that if to the square of each, the three remaining squares be added, the *four* sums shall be square numbers; or if the square of the sum of every two of them be increased by the two remaining squares, the *six* sums thence arising shall be all squares; or if the square of the sum of every three of them be added to the remaining square, the *four* numbers thus found shall be all squares.

By MR. SAMUEL JOHNSON.

If we take any four squares as x^2, y^2, z^2 and v^2 and make their sum equal $\frac{1}{4}$, we shall obtain squares which will satisfy the conditions of the question; viz. assume $x^2 + y^2 + z^2 + v^2 = a^2$, or $\dfrac{x^2}{4a^2} + \dfrac{y^2}{4a^2} + \dfrac{z^2}{4a^2} + \dfrac{v^2}{4a^2} = \dfrac{1}{4}$ are general expressions for the for the fraction $\frac{1}{4}$

Now since $x^2 = a^2 - y^2 - z^2 - v^2$, put $x = a - r$, then $(a - r)^2 = a^2 - y^2 - z^2 - v^2$; from which $a = \frac{r^2 + y^2 + z^2 + v^2}{2r}$.

Let $r = 1$, $y = 2$, $z = 3$, and $v = 4$; hence $a = 15$ and $x = a - r = 14$; hence $\frac{x^2}{4a^2}$ &c., are $\frac{196}{900}$, $\frac{16}{900}$, $\frac{9}{900}$, $\frac{4}{900}$ or the fractions reduced become $\frac{49}{225}$, $\frac{4}{225}$, $\frac{1}{100}$, $\frac{1}{225}$, the squares required.

Ingenious solutions are also given by Messrs. Ryley, Wright, Drake, Settle, Whitehead, Baines and Wilkinson.

(209) *By Mr.* S. JOHNSON.

Given the radius of the circumscribing circle and the radii of the two circles circumscribing the two triangles made by the line bisecting the vertical angle, to construct the plane triangle.

By Mr. JONATHAN CROWTHER, *Frodsham, Cheshire.*

The solution of this question may be facilitated by the following

LEMMA. Let ABC be any plane triangle and circumscribing circle, FE the vertical diameter. Join AF, BF; apply the chord CG = AF, cutting AF in Q, and CH = FB, cutting FB in R. Then will AQ, QF, or FR, RB be equal "the radii of the circles circumscribing the two triangles made by the line bisecting the vertical angle" ACB.

For, because the chords GC, AF, and consequently the arcs GC, AF are equal, taking away the arc GF, common to them both, we have arc AG equal arc FC, and (in like manner) arc BH equal arc FC. Therefore (Euc. 3, 27) $\angle$ QAC equal $\angle$ QCA equal $\angle$ RBC equal $\angle$ RCB, and the triangles AQC, BRC are isosceles, and similar to each other. Again, in the triangles QFC, RCF, because (Euc. 3, 27) the $\angle$s QFC, QCF are equal to the $\angle$s RCF, RFC respectively, and FC is common to both triangles, the triangles (Euc. 1, 26) are similar and equal, and FR $=$ QC $=$ AQ, also QF $=$ CR $=$ RB. Let QD, parallel to FB, meet AB in D; and join DR. Then QD equal AQ equal QC, and A, D, C are in the circumference of a circle whose centre is Q. Also, because FR is equal and parallel, QD, DR is also parallel to and equal QF equal BR equal RC, and D, B, C are in a circle whose centre is R.

Once more, by similar triangles, AC : BC :: AQ : BR :: AD : BD, and (Euc. 6, 3) the line joining CD bisects the $\angle$ ACB. Q. E. D. And we have the following

Const. With the given radius describe the circle circumscribing the whole triangle, and draw the vertical diameter FE. Apply the chords FA, FB equal the sum of the two remaining radii. Join AB; divide AB in D in the ratio of the two last mentioned radii; through D draw EDC meeting the circle in C; join AC, BC; and ABC is the triangle required.

The Demonstration is contained in that of the preceding Lemma.

By Messrs. Swale *and* Ryley.

Comp. Apply in the given circumscribing circle DA equal DB equal the sum of the given radii AQ, QD; and draw AB; also DE perpendicular to AB; to AB apply QP equal QA; through E,P draw EC; join A, C, B, C: the required $\triangle$ is ACB, Draw QP, also PR parallel to AD, meeting DB at R.

The $\triangle$ ACB is inscribed in the given circle; it only remains to shew that the circles described through the points A, P, C, and B, P, C have their radii equal those given.

That CPE bisects the vertical $\angle$ ACB is obvious, for by the Comp. AD equal DB, and DE perpendicular to AB, or the $\angle$ ADE equal $\angle$ BDE, and the circumference AEB is therefore bisected at E: that is, the $\angle$ ACP equal $\angle$ BCP.

Also AQ equal QP, or QP parallel to DB, then $\angle$ AQP equal $\angle$ ADB equal 2 $\angle$ ACP, that is, Q is the centre of the circle passing through the points A, P, C; having the given radius AQ or QP by Comp. In like manner, $\angle$ PRB equal $\angle$ ADB equal 2 $\angle$ PCB, or R the centre of the circle passing through the points B, P, C; having, because of parallels, RP equal RB equal DQ equal the other given radius by the Composition.

Draw QR; then will PC be bisected perpendicularly by QR. Also AQ : QD = AC : CB.

Ingenious geometrical constructions were also transmitted by Messrs. Wright, Waterhouse, Johnson, Stringer, Settle, Whitehead and Shenstone.

(210) *By Mr. R.* SMITH, *Manchester.*

At the game of Whist what are the odds that the side which deals has three honours or all four?

By Mr. S. RYLEY, *Schoolmaster, Leeds.*

(This is Problem 48th of De Moivre's Chances, where the odds are stated as 7 to 13 without the process.) Let us suppose the turned up trump to be an honour; then we must find the probability of taking two set cards in 25 out of a stock containing 51; amongst which are three of the set cards. Hence (by Problem

20th of the above work) the probability will be $\frac{3\times 26\times 25\times 24}{51\times 50\times 49}=\frac{12\times 26\times 3}{51\times 49}$.

But if the trump turned up be not an honour, then we must find the prob. of having 3 given cards in 25 out of a stock of 51, containing 4 of the given kind, which by the aforesaid problem will be $\frac{\frac{4}{1}\times\frac{3}{2}\times\frac{2}{3}\times 25\times 24\times 23\times 26}{51\times 50\times 49\times 48}$ $=\frac{26\times 23}{51\times 49}$.

But the probability of taking the three honours must be estimated before the turn-up card is known, and there being 4 chances in 13 for an honour to turn up, and 9 against it, therefore the 1st of the above found probabilities must be multiplied by $\frac{4}{13}$, and the 2nd by $\frac{9}{13}$, and their sum taken, viz. $\frac{12\times 26\times 3\times 4}{51\times 49\times 13}+\frac{26\times 23\times 9}{51\times 49\times 13}=\frac{96}{833}+\frac{138}{833}=\frac{234}{833}=$ probability of their having 3 honours precisely.

Lastly we must find, in like manner, the probability of the dealers having 4 honours, which is the same as Problem 48th above referred to, and is there found to be $\frac{115}{1666}$, which is one of the proposer's results in answer to the 174th Question of this work, which being added to the above, gives $\frac{234}{833}+\frac{115}{1666}$ $=\frac{583}{1666}$, the probability required.

Hence the odds are as 583 to 1083, or as 7 to 13 nearly, agreeing with De Moivre's statement.

By Mr. Wm. Settle

By the laws of the game, the probability of the dealer turning up an honour will be $\frac{4}{13}$, and the probability of drawing two or more set cards in 25 out of a stock of 51, will (by De Moivre's laws of chance

Problem 20th) be truly expressed by $1 - \left(\frac{26.25.24}{51.50.49} + \frac{3.26.25.25}{51.50.49}\right) = \frac{808}{1666}$; therefore the probability of the dealer and his partner having three or more honours in this case will be $\frac{808}{1666} \times \frac{4}{13} = \frac{3232}{21658}$.

Secondly the probability of the dealer not turning up an honour will be $\frac{9}{13}$, and by the above-cited problem, the probability of the dealer and his partner drawing three or more set cards in 25 out of a stock of 51, will be $1 - \left(\frac{26.25.24.23}{51.50.49.48} + \frac{4.26.25.24.25}{51.50.49.48} + \frac{6.26.25.25.24}{51.50.49.48}\right) = \frac{483}{1666}$.

Therefore the probability of the two dealers having three or more honours in this case will be $\frac{483}{1666} \times \frac{9}{13} = \frac{4347}{21658}$. Hence $\frac{3232 + 4347}{21658} = \frac{7579}{21658}$, the probability of the dealer and his partner having three or more honours; therefore the odds are 14079 to 7579, or 13 to 7 nearly, against the dealers.

⁂ Remark. The above is only one part of De Moivre's 48th Prob. where that learned Author asserts, that, by the help of the 20th Problem, this and many others of the like nature may be easily verified.

By Mr. JAMES WHITEHEAD.

It is obvious that for the side which deals, to have three honours or all four, one of the following events must happen:—viz. the card turned up must be a blank, and they must receive either three or four honours among the remaining 25 cards; or the card turned up an honour, and they must receive two or three honours among the remaining cards; but the probability of each of these events happening stands thus:—(1) $\frac{9}{13} \times \frac{4}{51} \times \frac{3}{50} \times \frac{2}{49} \times \frac{47}{48} \times \frac{46}{47}$ &c. … $\frac{26}{27} \times \frac{25.24.23}{1.2.3} = \frac{6.23}{17.49} = A$. (2) $\frac{9}{13} \times \frac{4}{51} \times \frac{3}{50} \times \frac{2}{49} \times \frac{1}{48} \times \frac{25.24.23.22}{1.2.3.4} = A \times \frac{11}{52}$ (3) $\frac{4}{13} \times \frac{3}{51} \times \frac{2}{50} \times \frac{48}{49} \times \frac{47}{48}$ &c.… $\frac{26}{27} \times \frac{25.24}{1.2} = \frac{16}{23}A$. (4) $\frac{4}{13} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49} \times \frac{25.24.23}{1.2.3} = \frac{8}{39}A$. therefore $1 - \frac{583}{1666} = \frac{1083}{1666}$ the probability of the contrary.

Hence the odds are as 1083 to 583, or as 20 to 11 nearly.

Ingenious solutions were received from Messrs. Wright, Drake, and the Proposer.

(211) *By* DIOPHANTUS.

It is required to find three positive cube numbers such that their sum shall be a square, and if the square of each be added to the sum of the other two, the numbers thence arising shall be all squares.

By MR. SAMUEL JOHNSON.

Let x^3, y^3, z^3 be the cubes required, assume $x^3 + y^3 + z^3 = a^2$; then $x^3 = a^2 - y^3 - z^3$; and $x^6 = (a^2 - y^3 - z^3)^2$; and $x^6 + (y^3 + z^3) = (a^2 - y^3 - z^3)^2 + y^3 + z^3$: now it is easily seen, if $a = \frac{1}{2}$ or $a^2 = \frac{1}{4}$, all the conditions except the first will be satisfied; hence it only remains to determine x, y, and z, when $x^3 + y^3 + z^3 = \frac{1}{4}$. Now the question will be more general if we put $x^3 + y^3 + z^3 = m^{2n}$, n being any integer.

First, by transposition, we have $x^3 = m^{2n} - y^3 - z^3$; assume $x = r - (y + z)$; then $x^3 = r^3 - 3r^2 (y + z) + 3r (y + z)^2 - (y + z)^3 = m^{2n} - y^3 - z^3$; which equation being ordered, gives $y^2 (3r - 3z) - 3y (r - z)^2 = m^{2n} - r^3 - 3rz^2 + 3r^2z = m^{2n} - z^3 - (r - z)^3$; hence by completing the square and taking the root, we have $y = \frac{r - z}{2} \pm \sqrt{m^{2n} - z^3 - (r - z)^3 + \frac{(r - z)^2}{4}} = \frac{r - z}{2} \pm \sqrt{\frac{4m^{2n} - 4z^3 - (r - z)^3}{12 (r - z)}} = \frac{r - z}{2} \pm \sqrt{\frac{[4m^{2n} - 4z^3 - (r - z)^3] \times 3 (r - z)}{6 (r - z)}}$.

Let $r - z = 3d^2$, then $y = \frac{3d^2}{2} \pm \frac{1}{6d} \sqrt{[4m^{2n} - 4r^3 + (6rd - 9d^3)^2]}$; hence it appears that $4m^{2n} - 4r^3 + (6rd - 9d^3)^2$ must be a square; assume its root $= 2m^n + (6rd - 9d^3)$, then by squaring $4m^{2n} + 4m^n (6rd - 9d^3) + (6rd - 9d^3)^2 = 4m^{2n} - 4r^3 + (6rd - 9d^3)^2$.

Now the first and last terms on each side destroy each other, and there remains $r^3 = m^n (6rd + 9d^3)$; make $fd = r$, then $f^3d^3 = m^n (9d^3 - 6fd^2)$; from which $d = \frac{6fm^n}{9m^n - f^3}$: hence $r = \frac{6f^2m^n}{9m^n - f^3}$; also $z = r - 3d^2 = \frac{6f^2m^n}{9m^n - f^3} - \frac{108f^2m^{2n}}{(9m^n - f^3)^2} = \frac{-54f^2m^{2n} - 6f^5m^n}{(9m^n - f^3)^2}$.

Now though this value of z appears to be negative, yet by using negative values for f, positive values may be found for it: also $y = \frac{3d^2}{2} \pm \frac{1}{6d}\left(2m^n + \frac{216f^3m^{2n}}{(9m^n - f^3)^2}\right) -$

$$\frac{9\times 216f^3m^{3n}}{(9m^n-f^3)^3}=\frac{54f^2m^{2n}}{(9m^n-f^3)^2}\pm\left(\frac{729m^{3n}-243m^{2n}f^3-81f^6m^n-f^9}{18f(9m^n-f^3)^2}\right)$$

by substituting $\frac{6fm^n}{9m^n-f^3}$ for d.

Now in the present case, $m=\frac{1}{2}$, and $n=1$; hence $z=\frac{-54f^2-12f^5}{(9-2f^3)^2}$, $y=\frac{54f^2}{(9-2f^3)^2}\pm\left(\frac{729-486f^3-324f^6-8f^9}{36f(9-2f^3)^2}\right)$; and if we take $f=-2$, we obtain $d=\frac{6fm^n}{9m^n-f^3}=-\frac{12}{25}$, $r=\frac{24}{25}$, and $z=\frac{12096}{72\times(25)^2}$, $y=\frac{27575}{72\times(25)^2}$, and $x=r-(y+z)=\frac{3529}{72\times(25)^2}$: hence $x^3+y^3+z^3=\frac{22781250000000}{91125000000000}=\frac{1}{4}$ as required.

We shall illustrate the use of the preceding formulæ by giving two more examples.

1. Let $m=2$, $n=1$, then $m^{2n}=4$, and $x^3+y^3+z^3=4$, and the expressions above become $z=\frac{-256f^2-12f^5}{(18-f^3)^2}$, also $y=\frac{216f^2}{(18-f^3)^2}\pm\left(\frac{5832-972f^3-162f^6-f^9}{18f(18-f^3)^2}\right)$; now let $f=-3$, then $r=\frac{262440}{54\ (45)^2}$, $z=\frac{52488}{54\ (45)^2}$, and $y=\frac{171315}{54(45)^2}$; hence $x=r-(y+z)=\frac{117369}{54(45)^2}$, and the numbers further reduced give $\left(\frac{53}{6\times(25)^2}\right)^3+\left(\frac{72}{6\times(25)^2}\right)^3+\left(\frac{235}{6\times(25)^2}\right)^3=4.$

2. Let $m = 3$, $n = 1$, then $m^2n = 9$, and $x^3 + y^3 + z^3 = 9$, and per above, $z = \frac{-486f^2 - 18f^5}{(27 - f^3)^2}$, also $y = \frac{486f^2}{(27 - f^3)^2} \pm \frac{729 \times 27 - 234 \times 9f^3 - 81 \times 3f^6 - f^9}{18f(27 - f^3)^2}$

Now taking $f = -6$, we obtain $z = \frac{18144}{12 \times (27)^2}$, $y = \frac{3645}{12 \times (27)^2}$, also $r = \frac{8}{3} = \frac{23323}{12 \times (27)^2}$, and $x = r - (y + z) = \frac{1539}{12 \times (27)^2}$; hence $x^3 + y^3 + z^3 = \frac{6025163444928}{669462604992} = 9$.

Hence it appears that every square number is capable of being divided into three cube numbers; however, I am of opinion much remains to be done on the subject of this important question, though what is here done may be a hint to those whose qualifications and leisure are better adapted to the subject than mine are at present.

By MR. SAMUEL RYLEY.

From the solution of the ninth question it appears that we have only to render the equation $\frac{1}{4} = x^3 + y^3 + z^3$ rational. Let $x = \frac{r}{2}$, $y = \frac{s}{2}$, and $z = \frac{v}{2}$ then $2 - r^3 - s^3 = v^3$. Let $r = \frac{n + b}{n + m}$ and $\frac{g}{n + m} = s$; therefore $n^3 + (6m - 3b).n^2 + (6m^2 - 3b^2)n + 2m^3 - b^3 - g^3 = v^3$.

Assume $v = n + (2m - b)$ then we find $n = \frac{12m^2b - 6mb^2 - 6m^3 - g^3}{6m^2 - 12mb + 6b^2}$. Let $g = (m - b).t$;

therefore $n = -m - \frac{m - b \times t^3}{6}$; but as this value of n will in general be negative, let $n = p - m - \frac{(m-b).t^3}{6}$ then $v^3 = p^3 + \left(3m - 3b - \frac{(m - b).t^3}{2}\right).p^2 + \left[6mb - 3m^2 - 3b^2 + (b - m).(m - b).t^3 + \frac{(m - b)^2.\, t^6}{12}\right] .v + \left(1 - \frac{t^3}{6}\right)^3 . (m - b)^3$.

Assume $v = sp + \left(1 - \frac{t^3}{6}\right) . (m - b)$, then we have $p^3 + \left(3m - 3b - \frac{(m - b).t^3}{2}\right. p^2 + (6mb - 3m^2 - 3b^2 + (b - m) \times (m - b).t^3 + \left.\frac{(m - b)^2.t^6}{12}\right).p = s^3p^3 + \left(1 - \frac{t^3}{6}\right).(m - b) . 3s^2p^2 + \left(1 - \frac{t^3}{6}\right)^2 . (m - b)^2 . 3sp$; hence by equating the coefficients of p we get $s = \left(\frac{t^6}{36} - \frac{t^3}{3} - 1\right) \div \left(1 - \frac{t^3}{6}\right)^2$ or $s = \frac{t^6 - 12t^3 - 36}{t^6 - 12t^3 + 36}$; and again, by equating the remaining terms of the equation we get $p = \frac{(6-t^3).(s^2-1).(m-b)}{2 \times (1 - s^3)} = \frac{(t^3 - 6)^2.(12t^3 - t^6).(6 - t^3).(m - b)}{3t^{12} - 72t^9 + 432t^6 + 1226}$: therefore $n = \frac{(t^3 - 6)^2.(12t^3 - t^6).(6 - t^3).(m - b)}{3t^{12} - 72t^9 + 432t^6 + 1296} - m - \frac{(m - b).t^3}{6}$; where m, b, and t may be taken at pleasure.

Example. Let $t = 3$, $b = 0$, and $m = 1$; then we find $r = \frac{25609}{37791}$, $s = \frac{36546}{37791}$, and $v = \frac{34853}{37791}$ hence $x = \frac{25609}{75582}$, $y = \frac{36546}{75582}$, and $z = \frac{34853}{75582}$ values that will answer.

Ex. 2. Let $t = -2$, $b = 2$, and $m = 1$; then $r = \frac{25813}{20932}$, $s = \frac{9762}{20932}$, and $v = \frac{5971}{20932}$; and $x = \frac{25813}{41864}$, $y = \frac{9762}{41864}$, and $z = \frac{5971}{41864}$; three other numbers which answer the question.

By MR. C. WATERHOUSE.

This question is to divide the square number $\frac{1}{4}$ into three cubes. Let $\frac{1}{4} = a^3 + b^3 + c^3$ and by multiplying both sides by $8d^3$ we have, $2d^3 = 8a^3d^3 + 8b^3d^3 + 8c^3d^3$, or to find three cubes equal to double a cube, and by trial we soon find $2 \times 9^3 = 11^3 + 7^3 - 6^3$, but as the last of these cubes is negative, we must remedy this by finding two affirmative cubes equal to $11^3 - 6^3$; now (by Emerson's Algebra, problem 76) there are $x = b + v$, $y = c - \frac{b^2}{c^2}v$, and $v = \frac{3bc^3}{b^3 - c^3}$; in this case we may take $c = -6$, and $b = 11$, then will $v = -\frac{7128}{1547}$, $x = \frac{9889}{1547}$, and $y = \frac{14676}{1547}$, so that $\left(\frac{9889}{1547}\right)^3 + \left(\frac{14676}{1547}\right)^3 + \left(\frac{7 \times 1547}{1547}\right)^3 = 2 \times 9^3$, and dividing by 18^3 we have $\frac{1}{4} = \left(\frac{9889}{27846}\right)^3 +$

$\left(\frac{14676}{27846}\right)^3 + \left(\frac{10829}{27846}\right)^3$ which three cubes answer all the conditions of the question.

⁂ The proposer divides the fraction $\frac{1}{4}$ into three cubes whose sides are $\frac{161}{630}$, $\frac{253}{630}$ and $\frac{548}{630}$, the smallest numbers we have received.

This question was also ingeniously answered by Messrs. Drake and Whitehead.

(212) *By Mr. W. Shillito.*

If the base and vertical angle of a plane triangle be given, the sum of one side and the perpendicular is the greatest possible when their included angle is bisected by the straight line drawn from the vertex to the centre of the circumscribing circle. Required a demonstration by the Elements of Euclid.

By the Proposer, and Mr. Jonathan Crowther.

Let ABC, a triangle of which the base AB and vertical angle ACB are given, be circumscribed by a circle whose centre is O; join CO; demit CD perpendicular to AB, and to the point C draw the tangent FG. The aggregate of AC and CD is a maximum when OC bisects the angle ACD, or since OC is perpendicular to FG, when the angle ACF is equal to the angle DCG. For let C′ be any other point in the circle; join CC′, upon which as a diameter describe the circle CPC′I cutting C′D′ in I, and AC′ produced in P, and join CI, CP; then because (Euc. 31, 3) the angle C′IC is a right angle, D′ICD is a parallelogram; therefore ID′ = CD.

Let CD, C′D′ meet the circle again in E, E′; and join AE, AE′, CE′. Since FG touches the circle at C, therefore (Euc. 32, 3) the angle FCA = CEA, and

$\angle$ GCE $=$ CAE; but (by Hypothesis) $\angle$ FCA $=$ GCE; therefore $\angle$ CEA $=$ CAE. Now (Euc. 13, 1 and 22, 3) $\angle$ CC′P $=$ CEA $=$ CAE, and (Euc. 21, 3) $\angle$ CC′E′ $=$ CAE′; but $\angle$ CAE′ is greater than CAE, therefore CC′E′ is greater than CC′P; consequently the angle C′CP, the complement of CC′P, is greater than C′CI, the complement of CC′E′, and therefore C′P is greater than C′I.

Again since APC is a right angle, AC is greater (Euc. 17 and 19, 1) than AP or AC′ $+$ C′P: therefore AC is greater than AC′ $+$ C′I: add CD $=$ D′I to each; then will AC $+$ CD be greater than AC′ $+$ C′I $+$ D′I, that is AC $+$ CD is greater than AC′ $+$ C′D′. Q.E.D.

Mr Crowther remarks, that the geometrical construction of this question is very safely locked up, nobody knows where, with that of the celebrated problem of *the trisection of an arc.*

By Mr. J. H. Swale, *Liverpool.*

Let AB be a given chord in the circle ACB to centre Q; draw CA, CQ, and CDE perpendicular to AB, making the angle ACQ $= \angle$ ECQ; then will AC $=$ CE: for CA, CE, making equal angles with the radius or diameter passing through C, must be equally distant from the centre, and, consequently, equal to each other. Upon this depends the truth of Mr. Shillito's theorem. Draw AP, EP, to any point P in the periphery; demit upon AB the perpendicular PL in EP; take EI $=$ ED; and draw AE; the intersection of AB, EP, being at V.

Besides the $\triangle$ ACE there can be no other isosceles $\triangle$ inscribed in the given circle, having one of its sides CE perpendicular to AB: therefore the perimeter of the $\triangle$ ACE is greater than that of any $\triangle$ whatever, having the base AE, and its vertical angle equal the angle ACE; or, which is the same, its vertex in the periphery ACE: that is AC $+$ CE is greater than AP + PE, or obviously, AC + CD greater than AP + PI, and, consequently, still greater than AP + PV; or still greater than AP + PL: that is AC + CD is the greatest, or a maximum.

Draw the diameter GH perpendicular and CK parallel to AB; and let R be any point whatever in GH.

Then AC + RK will be a maximum when CQ bisects the angle ACD, as in the above.

I think we may enunciate a theorem yet more general thus; let AHM be a semi-circle to centre Q, and R′K′ a right line any how given in position; draw AC, QC, to the point C in the periphery, and CK′ perpendicular to R′K′, meeting R′K′ in F, T, K′, and making FC = FT; then will AC′ + R′K′ be a maximum.

Solutions were received from Messrs. Johnson, Stringer and Settle.

(213) *By Mr. J. H. Swale.*

To divide a given arc into two parts, such, that if a given line be added to the versed sine of each part, the sum or difference of the squares of the compounded lines shall be equal to a given space.

By the Proposer.

Suppose the given arc AVB divided by the radius CV into the proposed parts AV,VB, having the versed sines DV,LV: make VT = the given line; draw AF parallel and TF perpendicular to CV; produce BL to AF at E; make FI = FE; join AI, EI; let CG, a perpendicular to AB, meet EI at K; draw KA, also CN, CP, perpendicular to CT, CK, meeting KE, KA, at NP.

Because, of parallels, AF = DT, and EF = LT, or $DT^2 + LT^2 = AF^2 + FE^2 = AF^2 + FI^2 = AI^2$ = a given square; or AI a given line: that is, the locus of I is a given circle to centre A. Again, $\angle$ VCK = $\angle$ PCN = $\angle$ PKN, or the points C,K,N,P, in the periphery of a circle; then, $\angle$ CPK = $\angle$ CNK = $\angle$ TIK equal half a right angle: and $\angle$ PCK being a right angle, of course the $\angle$ CKP = half a right angle; or $\angle$ CPK = $\angle$ CKP, and therefore CK = CP a given line: for CG,GA are given lines, and, because of parallels, GK = GA: the locus of N is therefore a given circle to diameter KP.

Let now the circles, which are the loci of the points I,N, be described,

Then since CT is given, CN,TI parallel, and the $\angle$ CNI given, it follows that IN is a given line.

We have therefore two circles given in position and magnitude, and a given point K, the intersection of one of their peripheries with the right line AQ, joining the centres A,Q of the given circles, to draw the right line KLN, making the interposed segment IN of a given length.

⁂ This is the well-known problem of Apollonius on Inclinations; to which I have pointed out a simple and elegant method of solution at question 197, page 140, in the last Correspondent.

Otherwise, by the same.

Suppose the arc AVB divided by the radius CV into the proposed parts AV,VB: make VT = the given line; draw BE,TF perpendicular and AEF parallel to CT; also EKI, making $\angle$ FEI = half a right angle, meeting a semi-circle on AB at K, and FT at I; draw CK,AI also CN perpendicular to CT, meeting IE produced at N.

It is evident that AF,EF are the sums of the versed sines of the arcs AV,BV and the given line VT, or $AF^2 + EF^2$ = a given square; and per the equal $\angle$s EIF, IEF we have EF = FI, or $AF^2 + FT^2$ = a given square, equal AI^2 (per 47. i.E.:) then AI^2 is given, or AI a given line. To the given centre A and radius AI, equal to the side of the given square, describe a circle which will be given in position and magnitude.

Again, since the points E,K are in the periphery of the given semicircle AEB, and the $\angle$ FEK = half a right angle, the $\angle$ AEK, and, of course, the point K will be given, and C being a given point, the line CK is therefore given in position and length: describe a circle upon CK to contain half a right angle, this circle will pass through N; for, per parallels, $\angle$ CNK $= \angle$ EIK, or half a right angle; that is, the circle passing through N is given in position and magnitude.

Moreover, since CT is a given line, and because of given angles CT : IN a given ratio, the line IN is therefore given.

Whence, we have two circles given in position and magnitude, and a given point K at the intersection of one of their peripheries with the right line passing through the centres of the given circles, to draw IN through K, making the segment IN, interposed between the peripheries of the given circles, equal to a given line.

This is the general problem of Apollonius on Inclinations, whose solution is given at page 140, Vol. III.

The case for the difference of the squares of AF, EK appears to be a solid problem, since it reduces to the following:—

Having a semi-circle and a right line parallel to its diameter given in position; to draw a right line through one extremity of the diameter, so that a segment of a given length shall be intercepted between the periphery of the semi-circle and the line given in position.

The application of the Conic sections to the determination of this case is well known, and needs not be added.

It was also algebraically answered by Mr. W. Settle.

(214) *Prize Question, by* AMICUS.

Alfred and Henry run at the same time, from the opposite sides of a street 100 feet in breadth: Henry moving parallel to the sides of the street whilst Alfred endeavouring to overtake him, runs twice as fast, and continues always in a right line between Henry and his (Alfred's) first position. How far, supposing they move uniformly, must Alfred run to overtake Henry, and what will be the nature of the curve he describes in the race?

By the Proposer.

Let AB represent the breadth of the street, G, H, any two cotemporaneous positions of Alfred and Henry, BH' perpendicular to AB the path of Henry, and AGH' the curvilinear path described by Alfred. Since by the question the points A, G, H, are in a straight line, and the curve AG has to the perpendicular BH the given ratio of 2 (m) to 1, therefore the fluxion of AG $= m \times$ fluxion of BH. Put AB $= 100 = a$, the abscissa AE $= x$, the ordinate GE $= y$, and BH $= z$: then by similar triangles AB : BH :: AE : GE $= y = \frac{zx}{a}$: consequently $y' = \frac{zx' + xz'}{a}$, and the fluxion of

$$\text{AG} = \surd(x'^2 + y'^2) = \sqrt{x'^2 + \left(\frac{zx' + xz'}{a}\right)^2} = mz':$$

hence $a^2x'^2 + (zx' + xz')^2 = a^2m^2z'^2$.

To find the relation of x to z in this complex fluxional equation independently of series, we proceed thus:—

Assuming $z' = wx'$, we get by substitution and division $a^2 + (z + wx)^2 = a^2m^2z^2$, whence $z + wx = am\sqrt{w^2 - \frac{1}{m^2}}$. (1) Taking fluxions and writing wx' for its equal z', we have $2wx' + xw' = \frac{amww'}{\sqrt{w^2 - \frac{1}{m^2}}}$, or dividing by $2w$, then $x' + \frac{w'}{2w}x = \frac{amw'}{2\sqrt{w^2 - \frac{1}{m^2}}}$. Assume $\frac{v'}{v} = \frac{w'}{2w}$, therefore $x' + \frac{v'x}{v} = \frac{amw}{2\sqrt{w^2 - \frac{1}{m^2}}}$.

Multiply by v, then $vx' + x'v' = \frac{amvw'}{2\sqrt{w^2 - \frac{1}{m^2}}}$. Whence

by Integration $v \times x = \text{F}. \frac{amw'}{2\sqrt{w^2 - \frac{1}{m^2}}} \times v$.

Now since $\frac{v'}{v} = \frac{w'}{2w}$, therefore $v = w^{\frac{1}{2}}$, and F. $\frac{amw'v}{2\sqrt{w^2 - \frac{1}{m^2}}} = \text{F}\cdot \frac{amv^2v'}{\sqrt{v^4 - \frac{1}{m^2}}}$. But the fluent of $\frac{v^2v'}{\sqrt{v^4 - \frac{1}{m^2}}}$ is an arc A of an equilateral hyperbola intercepted between the vertex and the extremity of the semi-diameter v, the semi-axis being $= \left(\frac{1}{m}\right)^{\frac{1}{2}}$ (Vide Art. 930, Maclaurin's Fluxions.) Consequently $v \times x = am \times \text{A}$, or $v = \frac{am\text{A}}{x} = w^{\frac{1}{2}}$: then from equation (1) we get $z = am\sqrt{w^2 - \frac{1}{m^2}} - wx$, or by substitution, $z = am\left(\sqrt{v^4 - \frac{1}{m^2}} - \text{A}v\right)$, and $y = mx \left(\sqrt{v^4 - \frac{1}{m^2}} - \text{A}v\right)$ the equation of the curve sought.

Now when $x = a$, and $m = 2$, then $v = 2\text{A}$, or we have to find an arc A of the equilateral hyperbola whose semi-axis is $= (\frac{1}{2})^{\frac{1}{2}}$, such that its double shall be equal to the corresponding semi-diameter v. Hence $A = .47713$, $v = .95426$, and $z = am\left(\sqrt{v^4 - \frac{1}{m^2}} - Av\right) = 61.151$, the double of which, or 122.302 feet, is the length of the curve AGH′, or the distance which Alfred must run to overtake Henry.

Remark. If Alfred, instead of moving in the right line AH *towards* Henry, were to move in the continuation of that line on or *from* Henry, at the same time that the latter moves from H′ in the direction H′B: then if Alfred run twice as fast as Henry, he will just describe the oval AG′VG″A while Henry runs through twice the distance H′B, and consequently the oval's length will be quadruple of the right line H′B.

By Messrs. WHITEHEAD *and* DRAKE.

Let A and B be the first positions of Alfred and Henry, and D and H their positions at any other time; draw CD perpendicular to AB, and put AC $= x$, CD $= y$, curve AD $= z$, and $n = 2$AB; then by similar △s $x : y :: \frac{1}{2}n :$ BH $= \frac{1}{2}z$, or $ny' = x'z'$: but $y'^2 = z'^2 - x'^2$, therefore $(x'z')^2 = n^2(z'^2 - x'^2)$; Assume $z = x + ax^3 + bx^5 + cx^7 + dx^9 + ex^{11} + fx^{13} + gx^{15} + hx^{17} + ix^{19} + kx^{21}$ &c. then will $z' = x'(1 + 3ax^2 + 5bx^4 +$ &c.) and $(x'z') = xx'(2 + 4ax^2 + 6bx^4 + 8cx^6$ &c.) —consequently $n^2(z'^2 - x'^2) = n^2x'^2(6ax^2 + x^4(10b + 9a^2) + x^6(14c + 30ab) + x^8(18d + 42ac + 25b^2) + x^{10}(22e + 54ad + 70bc) + x^{12}(26f + 66ae + 90bd + 49c^2) + x^{14}(30g + 78af + 110be + 126cd) + x^{16}(34h + 90ag + 130bf + 154ce + 81d^2) + x^{18}(38i + 182ah + 150bg + 182cf + 198de) + x^{20}(42k + 114ai + 170bh + 210cg + 234df + 121e^2) = (x'z')^2 = x^2x'^2 \{ 4 +$

$16ax^2 + x^4(24b + 16a^2) + x^6(32c + 48ab) + x^8(40d + 64ac + 36b^2) + x^{10}(48e + 80ad + 96bc) + x^{12}(56f + 96ae + 120bd + 64c^2) + x^{14}(64g + 112af + 144be + 160cd) + x^{16}(72h + 128ag + 168bf + 192ce + 100d^2) + x^{18}(80i + 144ah + 192bg + 224cf + 240de \}$ —: and by equating the coefficients of the homologous terms, we shall have $a = \frac{2}{3n^2}$, $b = \frac{2}{3n^4}$, $c = \frac{44}{7.9n^6}$, $d = \frac{410}{7.9^2n^8}$, $e = \frac{652}{9^2.11n^{10}}$, $f = \frac{13604}{9^2.21.11n^{12}}$, $g = \frac{281752}{9^3.7^2.11n^{14}}$, $h = \frac{278962}{9^3.7^2.11n^{16}}$, $i = \frac{47706436}{7^2.9^4.11.19n^{18}}$, $k = \frac{1010275108}{7^3.9^4.3.11.19n^{20}}$, ——

Now when $x = \frac{n}{2} = 100$, there results $z = x(1 + ax^2 + bx^4$ &c.) $= 100\{1 + .1666666666 + .0416666666 + .0109126984 + .0028246252 + .0007146114 + .0001776046 + .0000437654 + .00008330 + .0000026824 + .0000006818\left(1 + \frac{1}{4} + \frac{1}{16}\text{\&c.}\right)\} = 122.30209626$ = the required length of the curve.

From the above it appears that the relation of x and y cannot be expressed in simple algebraic terms, the curve is therefore transcendental.

In this last manner nearly the solution is given by Messrs Crowther, Ryley, Settle and Johnson, whose numerical results all precisely agree with the numbers found above. It was also answered by Mr. Joseph Shenstone.

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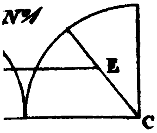

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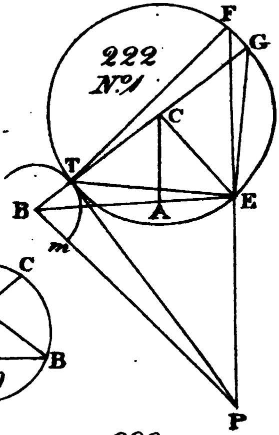

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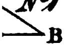

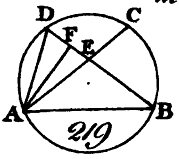

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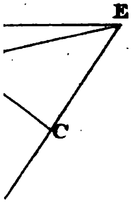

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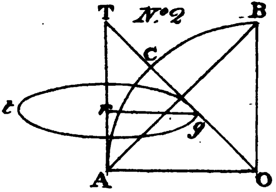

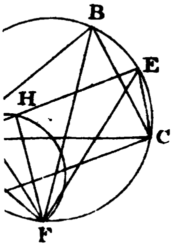

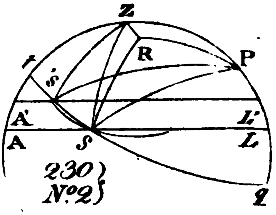

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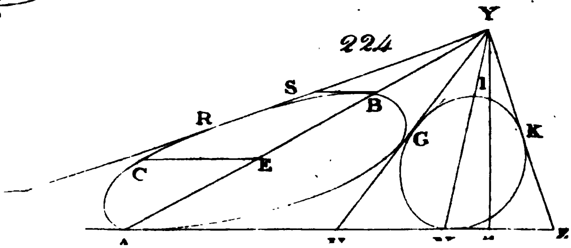

THE

LEEDS CORRESPONDENT,

A

Literary Mathematical, and Philosophical Miscellany.

Tarda fluunt ingrataque tempora, quæ spem
Consiliumque morantur agendi gnaviter id, quod
Æque pauperibus prodest, locupletibus æque;
Æque neglectum pueris senibusque nocebit.
HORAT.

No. IV. JULY, 1821. VOL. III.

JUVENILE DEPARTMENT.

ON RICHES.

A Translation of a French extract from the DICTIONARY OF THE PASSIONS.

ALL the gifts of fortune are generally called "riches;" but in this place we will look upon them as the possession of an infinite number of superfluous things, above a small number of those things which are really necessary. In this sense, do not riches turn aside those who possess them from the pursuit of truth? Do they not generally bring in their train corruption of manners?—The passages, which we are about to quote from Seneca, will serve to resolve those two questions, to which it does not appear that philosophers have hitherto paid much attention, although *they* are interested in them, more directly than *other men.*

"Riches have, in the case of many persons, been a great hindrance to philosophy. In order to enjoy that liberty of mind which is requisite for study, we must be poor, or must live like the poor. Every man who would lead a comfortable, quiet, and safe life, should avoid, as much as he can, those good things which are false and deceitful in appearance, and by whose allurements we suffer ourselves to be entangled as in a snare, without the power of subsequently releasing ourselves from it. In this respect we are the more unfortunate by thinking that WE possess them, and that, on the contrary, it is THEY which possess and tyrannize over us.

"Since riches have begun to be honourable amongst men, and have become, in some sort, the measure of public esteem, the taste for things truly fair and honourable has been entirely lost. So corrupted are we become, that we enquire, not of what utility a thing is, but what pleasure it affords. Our manners are so depraved, that we look upon poverty as a dishonour and a disgrace; in a word, that it is the object of the contempt of the rich, and of the hatred of the poor.

"Perpetual anxieties gnaw and devour the rich, in proportion to the wealth which they possess. Every one considers them happy men; every one would be in their circumstances. I confess it; but what an error! Is there any condition worse than to be incessantly exposed to misery and envy? Would to God, that those who seek after riches, would interrogate the rich upon their lot! Certainly they would in that case very soon cease to desire riches."

Let us reflect that he who speaks in these passages is a philosopher, who, according to Tacitus, possessed immense wealth; and then we shall perceive of what moment such a confession must be in his mouth.

All the merit of men appears, in these days, to be centred in riches, since they are become the universal passion. We place our highest felicity in becoming rich, and that only for our own satisfaction. We regard ourselves only in proportion to our expenditure: we consider as lost all that is not employed in furniture, buildings, and pleasure. "It is not," said Diogenes, "to obtain what would barely suffice for a subsistence upon herbs and fruits alone, that men seek to secure to themselves the government of a State, that they plunder cities, and wage war with foreigners, or even with their own countrymen; but they do these things, that they may partake of exquisite viands, spread out their tables with delicious meats, and have all the means of giving themselves up to debauchery."

"The Scythians," said Justin the historian, "despise gold and silver, as much as other men are enamoured of them; and it is to the contempt in which they hold these vile metals, as well as to their simple and frugal diet, that we must attribute the innocence and purity of their manners; because, being ignorant of riches, they have no occasion to covet the goods of another."—Zeno the Stoic (according to Seneca, Diogenes, Laertius, and other philosophers,) did not think more favourably of riches; for having learnt that the vessel on which he had embarked all his property, was shipwrecked, he said, without expressing any regret for this loss; "Providence designs that I should philosophize in greater tranquillity."

"Although riches," says Seneca again, "are very pretty presents, they are not what the wise man desires. He asks of heaven only what is simply necessary, with grace to use it properly. But riches are now become so necessary, they have acquired an importance so formidable, they procure for us all our wishes with so much ease, that it is not surprising if the multitude, more anxious to provide for their wants than to reflect on the source from which they arise, should worship them, believe nothing to be superior to them, and should blind themselves respecting the manner in which they are acquired. The wise man cannot think thus: he effaces, from the number of happy men, those who are only reputed happy on account of their wealth, and does not applaud the error of the populace, who generally account rich persons the most happy."

London. JOHN M. HARE.

ON HUMAN LIFE.

A translation from the French of Voltaire.

These verses, like many other compositions of the French Infidels, err greatly, by affording no bright relief to the dark shadows which the Poet has here depicted. It is religion alone that can produce such a relief. The believers in the revelation of God can understand how it is possible for HIM to cause '*all things to work together for their good;*' and, under the influence of Divine grace, they can '*rejoice evermore, pray without ceasing, and in every thing give thanks!*' This blessed state of mind, easy of attainment to all men, is vastly superior to that which is the subject of Voltaire's sombre description!

In these our fleeting days when righteous wrath
Bestrews with pain and grief our winding path,
Sons of one God, as brothers let us live,
And each fraternal aid to others give.
Beneath a weight of ills we lowly bend,
Our being curse, yet love it to the end.
The life we curse, yet cherish, num'rous foes
Delight to harrass and besiege with woes.
Our hearts unguided, friendless, and forlorn,
Burn with desire, or, listless, freeze with scorn:
Who can from sorrow's train exemption plead?
Wounded by grief, each human heart must bleed.
And though society's benignant spell
Might for a moment all our griefs repel,
That were too weak, too impotent a cure
For ills which we must constantly endure.
Ah! let us not that sweet repose destroy
Which still remains for mortals to enjoy!
Methinks I see, in dreary dungeon bound,
Some convicts laid along the humid ground,
Each might, if willing, ease the others' pains,
Instead of striking them with massy chains.

JUVENIS.

ON NATURAL DISPOSITIONS.

A Translation from the Latin of Valerius Maximus. (Lib. iii. Cap. 1.)

I will now just touch upon certain circumstances which serve to cherish the infancy of courage, and by which its very elements are developed. I will also notice the specimens of it given by a mind likely to attain in process of time to the highest pitch of glory, when its natural disposition manifests itself on some particular occasion.

Æmilius Lepidus, having entered into the army while yet a youth, killed an enemy, and preserved the life of a citizen. Of an achievement so celebrated there was a memorial placed in the Capitol, by a decree of the Roman Senate; it was a statue of him wearing the boss of his boyhood, and dressed in a cloak fringed with purple. For the Senate accounted it an act of injustice, if he who had arrived at this early maturity in courage, should on that account be deemed ineligible to receive public honours. Thus by the celerity he displayed in beginning early to act with bravery, Lepidus overstepped the disadvantages which proceeded from the immaturity of his years, and bore away double honours from the field of battle, of which his tender age had scarcely permitted him to be a spectator. For the hostile menaces of weapons, the clashing of drawn swords, the rapid quavering of hurled javelins, the clangour of cavalry entering on a charge, and the fearful impetus with which two armies commence an engagement, often contribute to strike a degree of terror even into the intrepid hearts of youthful warriors. But in the midst of these circumstances of war, the infant hope of the Æmilian family had sufficient valour to win the crown, and was able to bear away the spoils.

The same spirit displayed itself in the childhood of M. Cato. For when he was brought up in the house of Marcus Drusus his uncle, and the tribunes of the Latin people had assembled there for the purpose of obtaining the freedom of the city, he was asked by Poppedius, the chief of the Latins, who was likewise the guest of Drusus, to grant the allies (of whom the Latins were a part,) the benefit of his good offices with his uncle: but with a firm and unmoved countenance he replied, that he would not. After having been repeatedly importuned, he still persisted in this resolution. Poppedius having afterwards taken him up to the highest part of the house, threatened that he should be thrown down, if he would not comply with his request: but this danger could not move him from his purpose. Then did the whole of the assembly exclaim, "Let us Latins, and the whole of the allies, congratulate ourselves that he is so young. For if he had attained sufficient age to enter the Senate, we might then have ceased to entertain any hopes of gaining the privileges of Roman citizens." Cato therefore, while yet a child, had a clear perception of the dignified gravity which became the whole Senate, and by his perseverance repulsed the Latins when they were desirous of procuring from us a title to the rights of citizenship.

When the same youth came in his toga of rank to pay homage to Sylla, and saw the heads of those who had been proscribed brought into the hall, struck by the atrocity of the deed, he asked his tutor Sarpedon, why no one could be found to slay such a cruel tyrant. When the tutor had answered, that *it was not the will but the opportunity that was wanting to the people, because Sylla was safely defended by a strong military guard;* he earnestly entreated that a sword might be given to him, affirming that he could easily kill him, because he was accustomed to recline on his couch. The tutor complimented the courage of Cato, while he was shocked with his design, and never afterwards accompanied him to Sylla without having had him searched.—Nothing could be more admirable than this: The boy, when caught in this slaughter-house of cruelty, did not dread the tyrant, who was at the very time murdering consuls, municipal officers, the troops themselves, and the greater part of the Equestrian order. If any one had placed Marius himself in that situation, he would have meditated sooner upon a speedy flight, than have determined to kill the tyrant Sylla.

London. JOHN M. HARE.

IMMORTALITY OUGHT TO BE OUR OBJECT IN EVERY PURSUIT.

A Translation from the Latin of Pliny to his friend Paulinus, (Epist. iii. Lib. 9.)

Whatever any person may think of others, I account that man most happy who enjoys the foretaste of a good and lasting fame, and who, assured of the regards of posterity, lives in the anticipation of future glory. And indeed, unless the reward of eternity were presented to my view, that deep and fruitful leisure might afford me some delight. For I think it is incumbent on all men either to reflect on their immortality or on their mortality. Those who consider themselves immortal should incessantly aspire after it, and never be remiss; while others who view themselves only as mortal, ought to indulge in a state of quiet, and not burden or wear out a short life with toils which must soon cease. In this manner I behold several persons terminate their existence, under the pitiful and fruitless pretence of industry. These considerations on which I daily reflect, I leave with thee,—that if they do not gain thy approbation, I may cease to entertain them. But on this subject there will be no difference between us, because thou art one who art always meditating on something excellent and immortal. Farewell.

HOW GREATLY THE ADVICE AND APPLAUSE OF JUDICIOUS FRIENDS EXCITE MEN TO INGENIOUS STUDIES.

A Translation from the Latin of Pliny to his friend Mamilianus.
(Epist. xxv. Lib. 9.)

Thou complainest of the multitude of camp-business, yet, as if in the enjoyment of the most perfect leisure, thou dost read, admire and demand our plays and other trifling compositions, and art continually teazing me to write other pieces of this light description. For, since I received an opinion of them from thee, (who art a very wise, learned, and, in regard to the excellence of such writings, a most competent judge,) I begin to seek not only delight, but even glory, in studies of this kind. I am at present employed in legal pleadings, which, though not numerous, are pressing engagements; when these causes are heard, I will send some more of the same poetic effusions to thy kind and friendly bosom. Thou wilt impart wings to my little sparrows and young doves, and wilt allow them to soar among the eagles of your camp, provided their flight be agreeable to themselves and to thee; but if only pleasant to themselves, thou wilt then take care that they are confined to the dove-cote or the nest. Farewell.

JUVENIS.

POETRY.

THE BATTLE OF WATERLOO.

The following lines are the production of ISMAEL FITZADAM, "an able bodied seaman," (as the nautical phrase is,) whom our readers will recognise to be likewise ABLE-MINDED,—"heaven-born, and with the gift of song inspired." Several charming specimens of his poetical powers have lately appeared in the LITERARY GAZETTE; and a volume of his Poems has just made their appearance.

Oh for the poet's heart of flame!
A muse of fire for fiery theme!

To breathe one living, mighty lay,
Memorial worthy this proud day.
  Her steel of proof when Freedom drew,
Shook, like a sun-burst, red and high,
Her lion-standard to the sky,
Led on her sons to do or die,
  And struck, and won, at Waterloo!

Heroes! that bloody toil who shar'd,
For future glories haply spar'd,
Who wear, in brightest blazon wove,
Your victor-wreath—your country's love—
  And ye to whom 'twas given,
Sublimer lot! and worthier far!
To soar from that stern strife of war,
Like prophet borne on fiery car,
  On battle's blazing wing to heaven!

Ye brave, who stood, and ye, who fell,
Of those that fought such fields so well;
This day be ever vowed to you,
High meed to valour's memory due—
  And hark!—earth, sea, and sky,
In one wild, universal shout,
Proclaim the tyrant's final rout;
The cannon's thundering throat peals out,
  Stamping your fame's eternity!

Blest be their dust, who bled that day!
'Tis freedom's consecrated clay;
The mound that marks her martyrs' bones
The noblest far of earthly thrones!
  Oh! still, till time's last round,
As onward rolls the peaceful year,
Dear to the heart—to honour dear—
Religion's prayer, the patriot's tear,
  Shall mingle o'er that mound!

THE VALLEY OF FERN.

PART II.

From Poems by Mr. Bernard Barton, of the Society of Friends.

(Concluded from page 86.)

Thou art chang'd, lovely spot! and no more thou displayest,
To the eye of thy votary, that negligent grace,
Which, in moments the saddest, the tenderest, the gayest,
Allur'd him so oft thy recesses to trace.
The hand of the spoiler has fallen upon thee,
And marr'd the wild beauties that deck'd thee before;
And the charms, which a poet's warm praises had won thee,
Exist but in memory, and bless thee no more.
Thy green, palmy fern, which the softest and mildest
Of Summer's light breezes could ruffle,—is fled;
And the bright-blossom'd ling, which spread o'er thee her wildest
And wantonest hues,—is uprooted and dead.

Yet now, even now, that thou neither belongest,
Or seem'st to belong, unto Nature or Art;
The love I still bear thee is deepest and strongest,
And thy fate but endears thee the more to my heart.
Thou art passing away, like some beautiful vision,
From things which now *are*, unto those that *have been!*
And wilt rise to my sight, like a landscape elysian,
With thy blossoms more bright, and thy verdure more green.
Thou wilt dwell in remembrance, among those recesses
Which fancy still haunts; though they *were*, and *are not;*
Whose loveliness lives, and whose beauty still blesses,
Which, though ceasing to be, can be never forgot.

We know, all we see in this beauteous creation,
However enchanting its beauty may seem,

Is doom'd to dissolve, like some bright exhalation,
That dazzles, and fades in the morning's first beam.
The gloom of dark forests, the grandeur of mountains,
The verdure of meads, and the beauty of flowers;
The seclusion of valleys, the freshness of fountains,
The sequester'd delights of the loveliest bowers:
Nay, more than all these, that the might of old ocean,
Which seems as it was on the day of its birth,
Must meet the last hour of convulsive commotion,
Which, sooner or later, will uncreate earth.

Yet, acknowledging this, it may be that the feelings
Which these have awaken'd, the glimpses they've given,
Combin'd with those inward and holy revealings
That illumine the soul with the brightness of heaven,
May still be immortal, and destin'd to lead us,
Hereafter, to that which shall not pass away;
To the loftier destiny God hath decreed us,
The glorious dawn of an unending day.
And thus, like the steps of the ladder ascended
By angels (beheld with the patriarch's eye,)
With the perishing beauties of earth may be blended
Sensations too pure, and too holy to die.

Nor would Infinite Wisdom have plann'd and perfected,
With such grandeur and majesty, beauty and grace,
The world we inhabit, and thus have connected
The heart's better feelings with nature's fair face,
If the touching emotions, thus deeply excited,
Towards Him who made all things, left nothing behind,
Which, enduring beyond all that sense has delighted,
Becomes intellectual, immortal, as mind!
But they do; and the heart that most fondly has cherish'd
Such feelings, nor suffer'd their ardour to chill,
Will find, when the forms which inspir'd them have perish'd,
Their spirit and essence remain with it still.

Thus thinking, I would not recall the brief measure
Of praise, lovely valley! devoted to thee;
Well has it been won by the moments of pleasure
Afforded to some, justly valued by me.
May their thoughts and mine, often silently ponder
Over every lov'd spot that our feet may have trod;
And teach us, while through nature's beauties we wander,
All space is itself but the temple of God!
That so, when our spirits shall pass through the portal
Of Death, we may find, in a state more sublime,
Immortality owns what could never be mortal!
And Eternity hallows some visions of Time!

COMPARATIVE INTENSITY OF SOUNDS.

The inhabitants of Atures and Maypures, whatever the missionaries may have asserted in their works, are not more struck with deafness by the noise of the great cataracts, than the Catadupes of the Nile. When this noise is heard in the plain that surrounds the mission, at the distance of more than a league, you seem to be near a coast skirted by reefs and breakers. The noise is three times as loud by night as by day, and gives an inexpressible charm to these solitary scenes. What can be the cause of this increased intensity of sound in a desert, where nothing seems to interrupt the silence of nature? The velocity of the propagation of sound, far from augmenting, decreases with the lowering of the temperature. The intensity diminishes in air, agitated by a wind, which is contrary to the direction of the sound; it diminishes also by dilatation of the air, and is weaker in the higher than in the lower regions of the atmosphere, where the number of particles of air in motion is greater in the same radius. The intensity is the same in dry air, and in air mingled with vapours; but it is feebler in carbonic acid gas, than in mixtures of azot and oxygen. From these facts, which are all we know with any certainty, it is difficult to explain a phenomenon observed near every cascade in Europe, and which, long before our arrival in the village of Atures, had struck the missionary and the Indians. The nocturnal temperature of the atmosphere is 3° less than the temperature of the day; at the same time the apparent humidity augments at night, and the mist that covers the cataracts becomes thicker. We have just seen, that the hygroscopic state of the air has no influence on the propagation of the sound, and that the cooling of the air diminishes its swiftness.

It may be thought, that, even in places not inhabited by man, the hum of insects, the song of birds, the rustling of leaves agitated by the feeblest winds, occasion, during the day, a confused noise, which we perceive the less because it is uniform, and constantly strikes the ear. Now this noise, however slightly perceptible it may be, may diminish the intensity of a louder noise; and this diminution may cease, if during the calm of the night the song of birds, the hum of insects, and the action of the wind upon the leaves, be interrupted. But this reasoning, even admitting its justness, can scarcely be applied to the forests of the Oroonoko, where the air is constantly filled by an innumerable quantity of moschettoes, where the hum of insects is much louder by night than by day, and where the breeze, if ever it be felt, blows only after sunset.

I rather think, that the presence of the sun acts upon the propagation and intensity of the sound by the obstacles which they find in the currents of air of different density, and the partial undulations of the atmosphere caused by the unequal heating of different parts of the soil. In calm air, whether it be dry, or mingled with vesicular vapours equally distributed, the *sonorous undulation* is propagated without difficulty. But when the air is crossed in every direction by small currents of hotter air, the sonorous undulation is divided into two undulations; where the density of the medium changes abruptly, partial echoes are formed, that weaken the sound, because one of the streams comes back upon itself; and those divisions of undulations take place, of which Mr. Poisson has recently developed the theory with great sagacity. It is not therefore the movement of the particles of air from below to above in the ascending current, or the small oblique currents, that we consider as opposing by a shock the propagation of the sonorous undulations. A shock, given to the surface of a liquid, will form circles around the center of percussion, even when the liquid is agitated. Several kinds of undulations may cross each other in water, as in air, without being disturbed in their propagation; little movements may *ride over each other*, and the real cause of the less intensity of sound during the day appears to be the interruption of homogeneity in the elastic medium. During the day, there is a sudden interruption of density wherever small streamlets of air of a high temperature rise over parts of the soil unequally heated. The sonorous undulations are divided, as the rays of light are refracted, and form the *mirage* (looming,) wherever strata of air of unequal density are contiguous. The propagation of sound is altered, when a stratum of hydrogen gas is made to rise in a tube closed at one end above a stratum of atmospheric air; and Mr. Biot has well explained, by the interposition of bubbles of carbonic acid gas, why a glass filled with Champagne wine is little sonorous so long as the gas is evolved, and continues to pass through the strata of the liquid.

Humboldt's Personal Narrative.

MUSQUITOS.

Persons who have not navigated the great rivers of equinoctial America, for instance, the Oroonoko and the Rio Magdalena, can scarcely conceive, how without interruption, at every instant of life, you may be tormented by insects flying in the air, and how the multitude of these little animals may render vast regions wholly uninhabitable. However accustomed you may be to endure pain without complaint, however lively an interest you may take in the objects of your researches, it is impossible not to be constantly disturbed by the moschettoes, *zancudoes*, *jejens*, and *tempraneroes*, that cover the face and hands, pierce the clothes with their long sucker in the form of a needle, and, getting into the mouth and nostrils, set you coughing and sneezing whenever you attempt to speak in the open air. In the missions of the Oroonoko, in the villages placed on the banks of the river, surrounded by immense forests, the *plaga de les moscas*, "the plague of the flies," affords an inexhaustible subject of conversation. When two persons meet in the morning, the first questions they address to each other are, "How did you find the zancudoes during the night? How are we to-day for the moschettoes?" These questions remind us of a Chinese form of politeness, which indicates the ancient state of the country where it took birth. Salutations were made heretofore in the *celestial empire*, in the following words, *vou-tou-hou*, "Have you been incommoded in the night by the serpents?" We shall soon see, that on the banks of the Tuamini, in the river Magdalena, and still more at Choco, the country of gold and platina, the Chinese compliment on the serpents might be added to that of the moschettoes.

At Mandavaca we found an old missionary, who told us with an air of sadness, that he had spent *his* *twenty years of moschettoes* in America. He desired us to look well at his legs, that we might be able to tell one day, "poor alla (beyond the sea,) what the poor monks suffer in the forests of Cassiquiare." Every sting leaving a small darkish brown point, his legs were so speckled, that it was difficult to recognize the whiteness of his skin through the spots of coagulated blood. If the insects of the simulium genus abound in the Cassiquiare, which has *white waters*, the culices, or *zancudoes*, are so much the more rare; you scarcely find any there, while on the rivers of *black waters* in the Atabapo and the Rio Negro, there are generally some *zancudoes* and no *moschettoes*.

I have just shown, from my own observations, how much the geographical distribution of venomous insects varies in this labyrinth of rivers, with white and black waters. It were to be wished, that a learned entomologist could study on the spot the specific differences of these noxious insects, which in the torrid zone, in spite of their littleness, act an important part in the economy of nature. What appeared to us very remarkable, and is a fact, known to all the missionaries, is, that the different species do not associate together, and that at different hours of the day you are stung by a distinct species. Every time that the scene changes, and to use the simple expression of the missionaries, other insects "mount guard," you have a few minutes, often a quarter of an hour, of repose. The insects that disappear have not their places instantly supplied in equal numbers by their successors. From half after six in the morning till five in the afternoon, the air is filled with moschettoes; which have not, as we find related in some travels, the form of our gnats, but that of a small fly. They are simuliums of the family nemoceræ of the system of Latreille. Their sting is as painful as that of *stomoxes*. It leaves a little reddish brown spot, which is extravasated and coagulated blood, where their proboscis has pierced the skin. An hour before sun-set a species of small gnats, called *tempraneros*, because they appear also at sun-rise, take the place of the moschettoes. Their presence scarcely lasts an hour and a half; they disappear between six and seven in the evening, or, as they say here, after the *Angelus (a la oracion)*. After a few minutes' repose, you feel yourself stung by *zancudoes* another species of gnat (culex) with very long legs. The zancudo, the proboscis of which contains a sharp pointed sucker, causes the most acute pain, and a swelling that remains several weeks. Its hum resembles that of our gnats in Europe, but is louder and more prolonged. The Indians pretend to distinguish "by their song" the *zancudoes* and the *tempraneroes;* the latter of which are real *twilight insects,* while the *zancudoes* are most frequently *nocturnal insects,* and disappear towards sun-rise.

The *culices* of South America, have generally the wings, corselet, and legs of an azure colour, annulated, and variable from a mixture of spots of a metallic lustre. Here, as in Europe, the males, which are distinguished by their feathered antennæ, are extremely rare; you are seldom stung except by females. The preponderance of this sex explains the immense increase of the species, each female laying several hundred eggs. In going up one of the great rivers of America, it is observed, that the appearance of a new species of *culex* denotes the approximity of a new stream flowing in.

The whites born in the torrid zone walk barefoot with impunity in the same apartment, where a European, recently landed, is exposed to the attack of the *niguas* or *chegoes (pulex penetrans)*. These animals almost invisible to the eye, get under the nails of the feet, and there acquire the size of a small pea by the quick increase of its eggs, which are placed in a bag under the belly of the insect. The *nigua*, therefore, distinguishes, what the most delicate chemical analysis could not distinguish, the cellular membrane and blood of a European from those of a Creole white. It is not so with the moschettoes.

In the day, even when labouring at the oar, the natives, in order to chase the insects, are continually giving one another smart slaps with the palm of the hand. Rude in all their movements, they strike themselves and their comrades mechanically during their sleep. The violence of their blows reminds us of the Persian tale of the bear, that tried to kill with his paw the insects on the forehead of his sleeping master. Near Maypures we saw some young Indians seated in a circle and rubbing cruelly each others' backs with the bark of trees dried at the fire. Indian women were occupied with a degree of patience, of which the copper-coloured race alone are capable, in extirpating by means of a sharp bone the little mass of coagulated blood, that forms the centre of every sting, and gives the skin a speckled appearance. One of the most barbarous nations of the Oroonoko, that of the Ottomacs, is acquainted with the use of moschetto curtains *(mosquiteros)* formed of a tissue of fibres of the palm tree, *murichi*. We had lately seen, that at Higuerote, on the coast of Caraccas, the people of a copper-colour sleep buried in the sand. In the villages of the Rio Magdalena the Indians often invited us to stretch ourselves with them on ox-skins, near the church, in the middle of the *plaza grande*, where they had assembled all the cows in the neighbourhood. The proximity of cattle gives some repose to man. The Indians of the Upper Oroonoko and the Cassiquiare, seeing that Mr. Bonpland could not prepare his herbal, on account of the continual torment of the moschettoes, invited him to enter their ovens, *(hornitos)*. Thus they call little chambers, without doors or windows, into which they creep horizontally through a very small opening. When they have driven away the insects by means of a fire of wet brush-wood, which emits a great deal of smoke, they close the opening of the oven. The absence of moschettoes is purchased dearly enough by the excessive heat of stagnant air, and the smoke of a torch of *copal*, which lights the oven during your stay in it. Mr. Bonpland, with courage, and patience well worthy of praise, dried hundreds of plants, shut up in these *hornitos* of the Indians.

It is difficult not to smile at hearing the missionaries dispute on the size and voracity of the moschettoes at different parts of the same river. In the centre of a country ignorant of what is passing in the rest of the world, this is the favourite subject of conversation. "How I pity your situation!" said the missionary of the Raudales to the missionary of Cassiquiare, at our departure; "you are alone, like me, in this country of tigers and monkeys; with you fish is still more rare, and the heat more violent; but as for my flies, *(mia moscas)* I can boast, that with one of mine I would beat three of yours."

This voracity of insects in certain spots, the rage with which they attack man, the activity of the venom varying in the same species, are very remarkable facts; which find their analogy, however, in the classes of large animals. The crocodile of Angostura pursues men, while at Nueva, Barcelona, in the Rio Neveri, you may bathe tranquilly in the midst of these carnivorous reptiles. The jaguars of Maturin, Cumanacoa, and the isthmus of Panama, are cowardly in comparison to those of the Upper Oroonoko. The Indians well know, that the monkeys of some valleys can easily be tamed, while others of the same species, caught elsewhere, will rather die of hunger, than submit to slavery. I might have added the example of the scorpion of Camana, which it is very difficult to distinguish from that of the island of Trinidad, Jamaica, Carthagena, and Guayaquil; yet the former is not more to be feared than the *scorpio europœus* (of the south of France,) while the latter produces consequences far more alarming than the *scorpio occitanus*, (of Spain and Barbary.) At Carthagena and Guayaquil, the sting of the scorpion *(alacran)* instantly causes the loss of speech. Sometimes a singular torpor of the tongue is observed for fifteen or sixteen hours. The patient, when stung in the legs, stammers as if he had been struck with apoplexy.—*Ibid.*

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AVALANCHES OF THE CAUCASUS.

These immense falls are not confined to the winter season; but happen at any time, when either the power of the sun, or the weight of the snows, may disengage the preponderating load from its hold on the mountain. In June, 1776, the course of the Terek was stopped by one of these ice torrents; when its impeded water rose to the height of 258 feet, and suddenly tearing a passage through the rocky barrier of that tremendous defile, with a noise louder than thunder resounded by a thousand echoes, rushed onward in a devastating flood.

Similar was the horrid scene, report brought to us in the month of November, 1817. The pale summit of the mountain Kasibeck, on the side which shelves down into the dark valley between Derial and the village which bears the mountain's name, had been seen abruptly to move. In an instant it was launched forward; and nothing was now beheld for the shaken snow, and dreadful over-shadowing of the falling destruction. The noise that accompanied it, was the most stunning, bursting, and rolling onward, of all that must make death certain. As the avalanche rushed on, huge masses of rock rifted from the mountain's side, were driven before it; and the snows and ice of centuries, pouring down in immense shattered forms, and rending heaps, fell, like the fall of an earthquake; covering from human eye, villages, valleys, and people! What an awful moment, when all was still!—when the dreadful cries of man and beast were heard no more; and the tremendous avalanche lay a vast, motionless, white shroud on all around.

The magnitude of the destruction will readily be comprehended, when it is understood, that the depth of the snow, which thus rolled downwards in sight of the appalled inhabitants of the valley, was full twenty-eight fathoms, that is 168 feet; and its extent more than six wersts, or four miles, English. It immediately blocked up the course of the Terek, whose obstructed waters, beating up, in immense billows, foaming and raging against this strange impediment, seemed, at times, ready to overtop it; but, still repelled by the firmness and height of the snow, it fell back on its bed with a roaring that proclaimed the dreadful scene to a vast distance. The overcharged waters then formed themselves into a lake, which spread down the whole valley, on the river-side of its tremendous barrier; thus completely barring all communication with Wlady Caucasus. Nearly twelve days elapsed before the river had sapped its way through so immense a body of consolidated snow; but when it did make an opening, its flood, and fury, and devastating consequences, fell not far short of the dreadful ruin occasioned by the cause of its obstruction. Bridges, forts, every thing contiguous to its path, were washed away in the torrent.—*Porter's Travels.*

IMPERIAL PLOUGHING.

An Yu-she has accused the officers of Court for negligence at the Imperial ploughing match. It has long been a custom in China, derived from high antiquity, for the Emperor and the nobles of his Court, to go out into the fields, personally, once every year, and plough a certain piece of ground. The Emperor ploughs nine furrows, the princes of the Empire five, the nobles four, and so on in rotation, according to the various gradations of rank. At the late annual ploughing there appears to have been great negligence on the part of the nobles. When his Majesty had performed his share of the ceremony, he departed, instead of ascending the stage to witness the performance of his nobles, as was usual. The princes then did their part, and also departed. After which, the civil officers first dispersed, and the military officers, staring for some time at each other, ploughed but one furrow, and went away. "All which," the Emperor says, "was extremely negligent and improper. The princes, although they did no more than their duty, yet by their exactness in the performance of it, shewed that they had a respect for ancient ceremonies, for which they deserve praise and reward. But as to the nobles, if at the annual ploughing they could be thus careless and negligent, just because I did not personally inspect them—how much more sluggish and negligent will they be, when placed in offices of trust in various parts of the Empire: let them by no means escape severe punishment. Let the Censor who accused the military officers be rewarded by increase of rank—but where is the Censor who should have accused the civil officers? Let the nobles of the Court examine into this, and if it shall appear that the Censor of the civil officers was there witnessing the abuse of the ceremony, and neglected to report, let him be tried for the offence: but if it shall appear that he himself first decamped, and did not wait to see the ceremony performed, let him be more severely dealt with."

Indo-Chinese Gleaner.

DOMESTIC SLAVERY.

Canton, China, July 2, 1819.—This town has had its attention much excited for a few days past, by the discovery of a flagrant system of kidnapping male and female children.

The discovery was effected by the persevering efforts of a silk weaver, whose apprentice, the only son of a friend and benefactor, was kidnapped, shortly after entering on his master's service. The weaver walked the streets of Canton, in hopes of finding the lad, till, by his daily unsuccessful preregrinations, he was almost distracted. When all hopes of finding the youth had nearly forsaken his mind, he accidentally caught a glimpse of the object of his search. He recognized and spoke to the boy; but he, with a stupid gaze, affirmed, he knew nothing of the man. The weaver, however, carried him by force to his shop, and to the father of the lad, whose stupidity still remained, till the priests of Buddha and other religionists were called in to dispel the enchantment which had bereft the youth of his recollection. While various ceremonies were going on, the stupifying potion lost its influence, and the boy recognised with tears his master and his father.

The case was immediately communicated to the Government, by whom the rendezvous of the kidnappers was surrounded and all escape prevented. In the house were found six men and three women, who had for upwards of twenty years, carried on a successful system of kidnapping children, and sending them to other provinces to be sold for domestic slaves. They are said, during that period to have stolen several thousand children. Ten only unappropriated, were found in the house: and all, it is said, under influence of some infernal spell, or some narcotic drug administered to them. The kidnappers were put to the torture, under which two of the men died, and the remaining seven men and women, chained together, had been paraded about the streets of the city, receiving lashes as they went along, from the youths they had kidnapped. Whether they will be finally put to death, or transported, yet remains uncertain.

Were domestic slavery not allowed, kidnappers would have less inducement to practise their hateful occupation.—*Ibid.*

HOWARD'S TOMB.

The evening was drawing to a close when I approached the hill, in the bosom of which the dust of my revered countryman reposes so far from his native land. No one that has not experienced 'the heart of a stranger' in a distant country, can imagine the feelings which sadden a man while standing on such a spot. It is well known that Howard fell a sacrifice to his humanity; having caught a contagious fever from some wretched prisoners at Kherson, to whose extreme need he was administering his charity and his consolations. Admiral Priestman, a worthy Briton, in the Russian service, who was his intimate friend, attended him in his last moments, and erected over his remains the monument, which is now a sort of shrine to all travellers, whether from Britain or foreign countries. It is an obelisk of whitish stone, sufficiently high to be conspicuous at several miles' distance. The hill on which it stands, may be about three wersts out of the direct road, and has a little village and piece of water at its base. The whole is six wersts from Kherson, and forms a picturesque as well as interesting object. The evening having closed when I arrived at the tomb, I could not distinguish its inscription; but the name of Howard would be sufficient eulogy. At Kherson I learned that the present Emperor has adopted the plans, which the great philanthropist formerly gave in to the then existing government, for ameliorating the state of the prisoners. Such is the only monument he would have desired, and it will commemorate his name for ever; while that of the founder of the pyramids is forgotten—So much more imperishable is the greatness of goodness, than the greatness of power!—*Sir R. K. Porter's Travels in Persia.*

FRENCH MANAGEMENT.

At the aforementioned village, [Chaussèe] it seemed but too likely that we should procure nothing comfortable. But here we had a proof how well the French can make much out of little. A very small quantity of broth, which looked as if it had been saved from some feast of yesterday, was put on the fire, and to it were added a bit of cold boiled veal, a bit of previously dressed pork, and a remainder morsel of beef. A stew, which also seemed a remnant, was taken from a dish to be warmed up; and a little bit of veal was put down to roast. In Brittany, unlike the greater part of France, they roast extremely well. We had roast meat every day, as well cooked as we could have had it in England. In the sequel the dinner-table was set out with ample variety and due regard to appearances, though there was not a good dish upon it in point of quantity. The national taste for variety was however fully consulted.* There were the soup and bouilli, the fricandeaux and the roti; and afterwards the ceremony of the dessert was gone through with some stale cakes, and a few cherries.

The French bate nothing of externals in any situation. They have dinner arranged at the tables of the rich, and in the first hotels, according to the forms described; it follows also that the same sumptuous forms must be observed down to an hedge alehouse; the substantial merits of a dinner being abated at every stage, that the external magnificence may be wholly kept up. Gentlemen address each other with Monsieur, and therefore the postillions, carters, and peasants, do the same. In the Paris paper devoted to advertisements, I saw a cook-maid's application for a place in a kitchen; and those in need of one so qualified, were told to correspond with 'the young lady, Adele.' In England, at such a place as Chaussèe, the landlady of the inn would have put down but one dish of a homely kind, entreating her guests to excuse her humble fare; in France they never ask you to excuse any thing, but are always giving you occasion to accuse if you will.—*Scott's Sketches.*

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INDIAN RUBBER.

Here (at the mission of St. Balthasar on the Atobapo) we saw, for the first time, that white and fungous substance, which I have made known by the name of *dapicho* and *zapis.* We immediately perceived, that it was analogous to the *elastic resin;* but, as the Indians made us understand by signs, that it was found under ground, we were inclined to think, till we arrived at the mission of Javita, that the *dapicho* was a fossil caoutchouc, though different from the elastic bitumen of Derbyshire. A Poimisano Indian, seated by the fire, in the hut of the missionary, was employed in reducing the *dapicho* into black caoutchouc. He had spitted several bits on a slender stick, and was roasting them like meat. The *dapicho* blackens in proportion as it grows softer, and gains in elasticity. The resinous and aromatic smell, which filled the hut, seemed to indicate, that this coloration is the effect of the decomposition of a carburet of hydrogen, and that the carbon appears in proportion as the hydrogen burns at a low heat. The Indian beat the softened and blackened mass with a piece of brazil wood, ending in the form of a club; he then kneaded the *dapicho* into balls of three or four inches in diameter, and let it cool. These balls exactly resemble the caoutchouc of the shops, but their surface remains in general slightly viscous. They are used at San Balthasar in the Indian game of tennis, which is so celebrated among the inhabitants of Uruana and Encaramada; they are cut into cylinders, to be used as corks, and are far preferable to those made of the bark of the cork-tree.

Soon after, we obtained precise information respecting this substance:—it was shown us at the depth of two or three feet, in a marshy soil, between the roots of two trees known by the name of the *jacio* and the *curvana*. The first is the *hevea* of Aublet, or *siphonia* of the modern botanists, known to furnish the caoutchouc of commerce in Cayenne and the Grand Para; the second has pinnate leaves, and its juice is milky, but very thin, and almost destitute of viscocity. The *dapicho* appears to be the result of an extravasation of the sap from the roots. This extravasation takes place more especially when the trees have attained a great age, and the interior of the trunk begins to decay. The bark and alburnum crack; and thus is effected naturally, what the art of man performs to collect in abundance the milky juices of the hevea, the castilloa, and the caoutchouc fig tree.

The river Temi, near the banks of which this production is found in sufficient quantities to supply all Europe, runs through forests which overshadow it in so wild and luxuriant a manner as almost to mingle together the creatures of the several elements of air, earth, and water, and realize the classic images:

> Sæculum Pyrrhæ, nova monstra questæ;
> Omne quum Proteus pecus egit altos
> Visere montes;
> Piscium et summa genus hæsit ulmo,
> Nota quæ sedes fuerat Columbis,
> Et superjecto pavidæ natarunt
> Æquore damæ.

The Indians made us leave the bed of the river; and we went up towards the south, across the forest, through paths *(sendas)* that is, through open channels of four or five feet broad. The depth of the water seldom exceeds half a fathom. These *sendas* are formed in the inundated forest-like paths on dry ground. The Indians, in going from one mission to another, pass with their boats as much as possible by the same way; but the communications not being frequent, the force of vegetation sometimes produces unexpected obstacles. An Indian, furnished with a *machette* (a great knife, the blade of which is fourteen inches long,) stood at the head of our boat, employed continually in chopping off the branches that cross each other from the two sides of the channel. In the thickest part of the forest we were astonished by an extraordinary noise. On beating the bushes, a shoal of *toninas* (fresh water dolphins) four feet long, surrounded our boat. These animals had concealed themselves beneath the branches of a fromager or bombax ceiba. They fled across the forest, throwing out those spouts of compressed air and water, which have given them in every language the name of *blowers*. How singular was this spectacle in the middle of the land, three or four hundred leagues from the mouths of the Oroonoko and the Amazon! I am not ignorant, that the pleuronectes of the Atlantic go up the Loire as far as Orleans; but I persist in thinking, that the dolphins of the Temi, like those of the Ganges, and like the skate *(raia)* of the Oroonoko, are of a species essentially different from the dolphins and skates of the ocean. In the immense rivers of South America, and the great lakes of North America, nature seems to repeat several pelagic forms. The Nile has no porpoises: those of the sea go up the Delta no farther than Biana and Metonbis toward Selamoun.—*Humboldt's Travels.*

◆

To the Editor.

I send for your approval a translation of two interesting letters. The first is from Bessarion, Patriarch of Constantinople, who laid the foundation of the famous library of St. Mark at Venice. The sentiments which it contains will, on perusal, afford a mournful pleasure to every lover of polite learning, particularly in that part in which this profound scholar states his reasons for preferring Venice to any other city in Italy, as a place of safe custody for his invaluable collection of Greek and Latin books and manuscripts. His pious wishes for the increased stability and fame of that small Republic, have had the period of their accomplishment, till (like all mundane institutions,) the monuments of its pristine greatness have nearly passed away, and in the present age are doomed to exist only as records in the page of history. Venice suffered much from the French Republicans, in their early irruptions into Italy; and when, at the conclusion of the late war, the Great Powers of Europe divided the spoils of the world among themselves, it was on many accounts convenient to the House of Austria to embody in its territories the free states of Genoa and Venice, both of which, having been for ages fostered by the hand of freedom, are now drooping under the paralyzing arm of despotism.

The other, which is only an extract, ridicules the silly affectation of some learned men in making numerous quotations from books that have scarcely any reference to the subject on which they are writing.

But, without further preface, I introduce both of them to your notice.

VERAX.

LETTER I.

Bessarion, Patriarch of Constantinople, wisheth health to the most illustrious and invincible Prince, Christopher More, Doge of Venice, and to the honourable Senate of the Republic.

From à very tender age, I may almost venture to say "from my infancy," I have been accustomed to expend great labour, toil, care, and study in amassing as many books as I was able in every kind of learning. For this purpose both my childhood and my youth were devoted to the work of transcribing the greatest part of them with my own hand; and, at the same time, I disbursed, in the purchase of books, whatever sums of money a parsimonious frugality enabled me to save. For I thought I could not procure for myself any furniture for my house of greater value or magnificence than these, and no treasures more useful or excellent. Filled with the sayings and examples of the ancients, full of their manners, their laws and their religion, they live in our presence, hold communion with us, and talk in our company. They become our teachers, instructors and comforters; they represent to us things which had not left the slightest traces on our recollection, and place them immediately before our eyes. So great is their power, their dignity, majesty, and heavenly influence, that, were we destitute of books, we should all be changed into uncultivated and ignorant barbarians, retaining no remembrance of past events, deriving no benefit from preceding examples, and possessed of no knowledge concerning things human or divine. The very dust which covers up from vulgar gaze the bodies of men when they are interred, would then overwhelm and efface their names and achievements.

But though at all times I diligently applied myself to this occupation of collecting books, yet it was after the destruction of the Grecian empire and the horrid seige of Constantinople that I employed myself with still greater ardour and desire in closely searching for books written in the Greek language: in the attainment of these treasures I refrained from no pains, labour, or care; on this object I industriously spent all the strength of my powers and faculties. For in all my researches, I feared exceedingly lest, with the destruction of other things, such a great number of most excellent authors, the toils and watchings of such eminent men as were truly the lights of the world, might in a little time be placed in jeopardy or destroyed. In former times this had been the case, when we suffered a great loss; for, out of two hundred and twenty thousand volumes, which, according to Plutarch, were placed in the public library of Apamea, the capital of Phrygia, scarcely one thousand remain at the present time. But I endeavoured, in as judicious a manner as possible, to select books, not in reference to their bulk or the multitude of them, but in reference to their superior excellence and quality. I chose the best books of every description, and completed each set by obtaining all the volumes of which the work consisted. In this manner I collected almost all the productions of the wise Greeks, and in particular those which were of rare occurrence or difficult to be found.

But when I had frequently revolved all these circumstances in my mind, I seemed not to have fully met my own wishes unless I had likewise made provision that the books collected by me with much labour and study, should be deposited during my life-time in such a situation as might prevent at my decease their dispersion or transfer, and should be preserved in some place at once safe and convenient for the common benefit of learned men, both Greeks and Romans. While I was reflecting on this matter, and considering in my mind the advantages of the several towns of Italy, your famous and noble city was the only one which occurred to me as answering in every respect the views which I had formed.

For I could not perceive how I could make choice of any city of greater SAFETY, than that which being ruled with equity and restrained by wholesome laws, is under the guidance of union and wisdom; in which the virtues of continence, seriousness, justice and fidelity, have found an abode; in which the government is as distinguished for greatness and splendour, as it is for impartiality and moderation; the inhabitants of which evince liberality of mind in giving counsel, and are addicted to no sensuality or wickedness; and in which men of prudence guide the helm of state, good men are promoted in preference to the wicked, and, forgetful of their own private advantage, they combine together in managing the public concerns of the Republic with the greatest unanimity and integrity.—From these considerations, your city, it is hoped, will daily increase in stability and fame: in the warmest wishes for its prosperity I most heartily concur.

I also understood that I could not have selected any place more CONVENIENT than Venice, or better adapted to the needs of the inhabitants of Italy. Besides, persons from nearly every nation on the face of the earth assemble in your city, and, above all, the Greeks, who touch first at Venice when they come by sea from different provinces. They are bound to you by such close ties of amity, that when they approach your ports they seem but to be steering into another Constantinople. In this view, how could this benefaction be made with greater propriety than to those men to whom I am bound by a variety of benefits, and attached by the numerous favours which they have conferred on me? What city could I in this manner more appropriately endow than that which I chose for my home, when Greece, my own unhappy country, was subjugated by foreign conquerors, or to that city to which I was humanely invited by you, and in which I experienced a most honourable reception? Conscious therefore of my mortality, and considering that old age is daily gaining ground, and that I am already afflicted with several disorders, and may yet be attacked by many more, I have resolved to present all the books which I possess, in both languages, as a deed of gift to the most holy church of St. Mark in your famous city. To this act I am prompted by a degree of feeling which is due to your Excellencies, to my own gratitude, and to the country of which you wish me to be considered a denizen in common with yourselves. Since you by your virtues, wisdom and many special favours, have a strong hold on my affection, gratitude and esteem, I do this that you, your children and posterity may be the inheritors of my labours, and may derive from them every day the richest fruits in abundance; and that all others who are attached to sound learning, may on your account partake of the same advantages.

I therefore now transmit to your Excellencies this my donation, a catalogue of the books, and the decree of the Pope; accompanying them all with my prayers to God, that he would be pleased to grant to your Republic all happy and prosperous occurrences in tranquility, peace, public security and perpetual domestic concord.—Farewell to your Excellencies.

Viterbo, 12*th May*.

LETTER II.

Francis Petrarch to his Friend Thomas, of Messina.

* * * * * *

I have observed in thy writings how laborious thou art, and that, to all appearance, thou dependest entirely on thy course of study. From hence arises that wandering from one unknown volume to another, from each of which thou makest a selection to intersperse among thy own matters. For this thou receivest the applause of thy pupils, who, being astonished at the names of authors innumerable, call thee *omniscient*, as if thou hadst acquired a knowledge of all those books of whose titles thou retainest some recollection. But the learned quickly discern what passage is appropriately quoted and what is inapplicable,—what is borrowed, stolen, or dependent on the pleasure of another person,—what portion is drawn from the middle of a work and what is picked up in a cursory perusal. For a man to make a pompous display of his memory, is the glorying of a child; as Seneca says, "it is shameful for a man to employ himself in plucking buds," because it becomes his years to rejoice, not in flowerets, but in fruits. Thou art venerable in this part of thy age and of great celebrity in thy profession; and, instead of always complaining, that I may occasionally appear, to bestow due applause, let me add, in our days thou art without any rivalry an undisputed chief in that species of literature to which thou art indebted for thy fame; yet, with I-know-not what sort of a boyish disposition, thou wanderest beyond thy boundaries, and, quite at ease, thou saunterest about at the close of thy day in pastures belonging to other people, wasting thy time in picking up a few little buds in one corner and a few in another. Thou takest delight in attempting the discussion of things unknown; in which, not finding thy way quite clear, thou often either goest astray or tumblest down headlong. Thou also amusest thyself with treading in the footsteps of those persons who expose their knowledge as a sort of merchandise before their door, when in the mean time the house is quite empty. It is undoubtedly thy duty to strive in preference to be somewhat greater than thou now seemest to be. Boasting is always tiresome and full of danger. Suffer me likewise to give thee another intimation:—when thou shalt wish to appear as some great one, innumerable circumstances will occur, not only to reduce thee to thy real dimensions, but to contract thee much below the right standard. It is quite sufficient for one genius to obtain a glorious reputation in a single line of study. Those who pride themselves on the knowledge of many arts, are either truly divine personages, impudent fellows, or raving madmen. What man, among all the learned Greeks or Romans, is any where reported to have manifested such a species of presumption? But it is the modern practice, the temerity of these days, to make a vast display of vain-glorious inscriptions and professions, on the faith of which, as Pliny says, you might almost neglect to have the terms of a bond fulfilled, or might venture to forfeit your recognizances in a Court of Justice: But when you have passed these outward attractions and entered the house, what nothingness will you find withinside the dwelling! If therefore thou art inclined to give any credit to what I have said, let me persuade thee to confine thyself within thine own limits; and refuse to be an imitator of those who, while they promise all things, perform nothing, and who, under a pretence of meddling with every thing, and comprehending it, for that very reason bring themselves to understand nothing. This ancient Greek proverb contains much wholesome counsel:—"Let every one employ his powers in that art with which he is most familiarly acquainted."—Farewell.

SINGULAR CEREMONY.

As it is customary in Europe and other parts of the world to have solemn religious processions from time to time, on remarkable occasions, such as great victories, marriages of sovereigns, &c. we have here, (Washington, America,) for the purpose of making an impression on the minds of the people, contrived a procession of a different kind, which is intended as a kind of representation of Agriculture, Navigation, and Trade, as the great sources of national prosperity.

On the 15th of March there was a very singular procession of the butchers of this place. A few days before, it was announced by advertisement that the number of 67 oxen, 19 sheep, 4 bears, 3 deer, 10 goats, and 2 pigs, remarkably large and fat, were to be seen in a stable just without the city. These animals being slaughtered on the 13th and 14th, the carcases were loaded in above 100 carts, each drawn by one horse, adorned with ribbons and flowers, and carried in procession through the city; first went a large waggon drawn by four horses, carrying a band of musicians; over their heads was a kind of stage, in which stood the hide of the fattest ox stuffed with straw, looking as if alive; then came above 300 butchers on horseback, dressed in white, and decorated with blue silk ribbons; between them were the waggons with the meat, covered with white cloths; then came a waggon in which coopers were employed in making casks, and one in which butchers were cutting meat and making sausages; behind these followed a number of farmers all on horseback, with ears of corn in their hats; between them were ploughs, harrows, and other implements of husbandry drawn by horses; after those came a waggon drawn by 6 horses, on which was a ship with three masts completely rigged; in this vessel there was a company of musicians, and it was also manned with sailors, and bore the flag of the United States; then came a boat, also in a waggon; the procession was closed by a waggon bearing a fishing boat, manned with fishermen, with their implements. The procession consisted of above 1000 men on horseback, and passed through all the principal streets; it lasted from ten in the morning till four in the afternoon, and was concluded by a handsome dinner and a ball. The whole went off in the best order.

LUDICROUS SUPERSTITION.

When Lord *** came to an audience to take leave of his Neapolitan Majesty on his return to England, the King told him he had a little *bouquet d'adieu* for him; and when his Lordship probably dreamed of a gold snuff-box with the royal face set round with brilliants on the lid, he was presented with the heron's foot, as a spell against all accidents in an English fox-chase, and a remembrance of royal friendship and Neapolitan field sports.

ANSWERS

TO THE

QUESTIONS FOR YOUTH,

Proposed in the Third Number of the Third Volume.

(1) *By* Master *W.* PICKUP.

Cupid complained to his mother, that the Muses had taken away his apples: Clio, saith he, hath taken away $\frac{1}{5}$; Euterpe $\frac{1}{12}$; Thalia $\frac{1}{8}$; Melpomene $\frac{1}{20}$; Erato $\frac{1}{7}$; Terpsichore $\frac{1}{4}$; Polymnia 30; Urania 105; and Calliope, the most spiteful of them all, hath taken 360; so that I have only 5 apples left. How many apples had he at first?

By Master CHARLES MACAULEY.

Let $840x =$ the whole number of apples.

Then $168x$, $70x$, $105x$, $42x$, $120x$, and $210x =$ what each took respectively; and their sum $715x + 500 = 840x$ as by the question: whence $x = 4$, and $840x = 3360$, the sum required.

By Master RICHARD MOUNSEY.

Let $x =$ the number of apples.

Then $\frac{1}{5}x + \frac{1}{12}x + \frac{1}{8}x + \frac{1}{20}x + \frac{1}{7}x + \frac{1}{4}x + 500 = x$, or $\frac{143}{168}x + 500 = x$, or $x = 3360$, the apples required.

By Master GEORGE ASPDEN.

Per question $\frac{1}{5} + \frac{1}{12} + \frac{1}{8} + \frac{1}{20} + \frac{1}{7} = \frac{143}{168}$, then $1 - \frac{143}{168} = \frac{25}{168}$.

Again, $30 + 105 + 360 + 5 = 500$.

Whence, as $25 : 168 :: 500 : 3360$ the number of apples required.

(2) *By* Master *W.* BRAMWELL.

Four men, A, B, C and D jointly complete a piece of work in 12 days. A can do it alone in $\frac{3}{4}$ of the time that B can, B in $\frac{2}{3}$ the time of C, and C in $\frac{1}{2}$ the time of D. In what time can each separately perform the work?

By Master SAMUEL DOWSE.

By the question the ratios of their respective labours are $4 + 3 + 2 + 1 = 10$.

Hence, by Inverse Proportion,

$$\text{As } 10 : 12 :: \left\{ \begin{array}{lll} 4 : 30 \text{ days.} & \text{A's time.} \\ 3 : 40 \text{ ——} & \text{B's time.} \\ 2 : 60 \text{ ——} & \text{C's time.} \\ 1 : 120 \text{ —} & \text{D's time.} \end{array} \right.$$

By Master W. PICKUP.

Put $x =$ the time in which D will perform the work.

Then C can perform the work in $x \div 2$, B in $x \div 3$, and A in $x \div 4$.

$$\left. \begin{array}{rcccccc} & \text{d.} & \text{w.} & & \text{d.} & & \text{w.} \\ \text{As } x & : & 1 & :: & 12 & : & 12 \div x \\ x \div 2 & : & 1 & :: & 12 & : & 24 \div x \\ x \div 3 & : & 1 & :: & 12 & : & 36 \div x \\ x \div 4 & : & 1 & :: & 12 & : & 48 \div x \end{array} \right\} \begin{array}{l} \text{will express part of} \\ \text{the work done by A,} \\ \text{B,C and D in 12 days.} \end{array}$$

Then $(12 \div x) + (14 \div x) + (36 \div x) + (48 \div x) = (120 \div x) = 1$; therefore $x = 120$ days, the time in which D can perform the work alone; whence A can do it in 30, B in 40, and C in 60 days.

(3) *By* Master *RICHARD MOUNSEY.*

The sides of a right angled triangle are in Arithmetical progression, and 5 times the area is equal to the cube of the perpendicular from the right angle upon the hypothenuse. Required the sides and area.

By Master C. BARTHOLOMEW.

Suppose the sides to be $3x$, $4x$, and $5x$.

Then $12x \div 5$ will be the perpendicular. Per question $30x^2 = 1728x^3 \div 125$, or $1728x = 3750$, and $x = 3750 \div 1728$. Therefore the sides are $6\frac{49}{96}$, $8\frac{49}{72}$, and $10\frac{245}{288}$. Answer.

By Master THOMAS FAWCETT.

Let $x =$ one of the sides, and $y =$ the common ratio.

Then x, $x + y$, and $x + 2y$ will be the three sides; $\therefore$ by the question $x^2 + x^2 + 2xy + y^2 = x^2 + 4xy + 4y^2$ $\therefore$ $x^2 - 2yx = 3y^2$; and $x = 3y$, or $y = x \div 3 =$ the ratio.

Substituting this value of y in the expressions for the sides, they become x, $4x \div 3$, $5x \div 3$; then $(4x^2 \div 3) \div 2 = 2x^2 \div 3 =$ area of the triangle; and $(4x^2 \div 3) \times \frac{3}{5} = 4x \div 5 =$ the perpendicular from the right angle upon the hypothenuse; then by the question $10x^2 \div 3 = 60x^3 \div 125$: $\therefore$ $x = 1250 \div 192 = 6\frac{49}{96}$: whence $6\frac{49}{96}$, $8\frac{49}{72}$, and $10\frac{245}{288}$ are the three sides, and 28.2570165 is the area.

(4) *By* Master *JOHN WADDINGTON.*

One fourth of a certain number is equal to the difference of the squares of its two digits, and if the number be added to three fourths of itself, the digits will be inverted. Required the number.

By Masters T. LLOYD, J. FAIRCLOUGH, and HENRY WARD.

Let x and $y =$ the two digits.

Then by the question, $(10x + y) \div 4 = y^2 - x^2$ and $10x + y + (3x + 3y) \div 4 = 10y + x$, from this $y = 2$; substituting this value of y in the first equation, $x = 1$; $\therefore$ $y = 2$, whence the number is 12.

By Master W. HEBBLETHWAITE.

Let x represent the digit in the ten's place, and y that in the unit's place. Then by the question

$$(10x + y) \div 4 = y^2 - {}^2$$

$$10x + y + [3\,(10x + y)] \div 4 = 10y - x$$

From the 2nd equation $40x + 4y + 30x + 3y = 40y + 4x$, from which $y = 2x$. This value of y substituted in the 1st equation gives $x = 1$; therefore 12 is the number required.

(5) *By* Master *G. Aspden.*

The base, diameter and altitude of a right cone are each 9 inches. It is required to divide it into two parts in the ratio of 2 to 3 by a section parallel to the base.

By Master William Armitage.

Here $9^2 \times .7854 = 81 \times .7854 = 63.6174$ the area of the base: then $63.6174 \times \frac{9}{3} = 190.8522$ the solidity of the cone. Hence,

$$\left.\begin{array}{l} \text{As } 5 : 2 :: 190.8522 : 76.34088 \\ \text{As } 5 : 3 :: 190.8522 : 114.51132 \end{array}\right\} \text{the two parts.}$$

Supposing the top part to be the least;

As $190.8522 : 9^3 :: 76.34088 : 291.6^3$. and $\sqrt[3]{291.6} = 6.63125$ inches, the distance from the top that the section must be made.

By Masters William Charnley, Thos. Boardman, and George Ward.

Similar cones being as the cubes of their altitudes we shall have, As $\sqrt[3]{5} : \sqrt[3]{2} :: 9 : 9\sqrt[3]{\frac{2}{5}} = 9\sqrt[3]{.4} = .7368063 \times 9 = 6.6312567$ the height of the upper part and diameter. Or thus $9^3 \times .2618 = 190.8522$ the solidity of the whole cone, and

As $5 : 2 :: 190.8522 : 76.34088$ the solidity of the top part.

As the altitude and diameter will always be equal, the $\sqrt[3]{[76.34088 \div .2618]} = \sqrt[3]{291.6} = 6.6312567$ the height as before.

(6) *By* Master *C. Bartholomew.*

Supposing the earth to be a hollow globe of 8000 miles in diameter: what must be the thickness of the shell so that it may just float in air of the same density as that at its surface, the cavity being supposed a perfect vacuum, and the specific gravity of the shell double that of water?

* * *

By Master SAMUEL DOWSE.

Here 8000 miles are equal to 42240000 feet; then $[42240000]^3 \times .5236 = 39461298.005606400000000$ cubic feet, the solidity of displaced air which multiplied by $1\frac{1}{4}$ gives 49326622507008000000000 ounces, the weight of displaced air.—Now the weight of the metal of which the shell is composed, must be of equal weight with the displaced air. Consequently the weight of the displaced air divided by 2000 the specific gravity of the metal, gives 24663311253504000000 the cubic feet of metal contained in the shell, which, subtracted from the solidity, leaves 39436634694352896000000 cubic feet, the solidity of the vacuum; hence $\sqrt[3]{[39436634694352896000000 \div .5236]} = 42231198$ feet, the diameter of the vacuum.

Therefore $(42240000 - 42231198) \div 2 = 4401$ feet, the thickness of the shell required.

By Master HUGH HUNTINGTON.

Let 8000 miles $= 42240000$ feet $= a$, and the thickness of the shell $= x$, put also the specific gravity of the shell $= 2000 = S$, and that of air $= 1\frac{2}{8} = s$, and $.5236 = n$; then $na^3 =$ the solidity of the earth, also $a - 2x =$ the diameter of the cavity, and $n(a - 2x)^3 =$the content thereof:—hence $n[a^3-(a - 2x^3)]$ the content of the shell, and $Sn\,[a^3-(a-2x)^3] =$ the weight; But the weight of air displaced by the earth $= sna^3$; consequently $Sn \{ a^3 - (a - 2x)^3 \} = sna^3$, which gives $x = a \times \{ \sqrt[3]{S} - \sqrt[3]{(S - s)} \} \div 2\sqrt[3]{S}$, or by restoration $= 8000 \times \{ \sqrt[3]{2000} - \sqrt[3]{(2000 - 1\frac{2}{8})} \} \div 2\sqrt[3]{(2000)} = .8223$ of a mile, the required thickness of the shell.

By Master RICHARD MOUNSEY.

Put $a = 8000$ miles $= 42240000$ feet, the diameter of the globe, $b = 2000$ ounces, the weight of a cubic foot of the shell, $c = 1\frac{2}{9}$ the weight of a cubic foot of air, $p = 5236$, and $x =$ the diameter of the vacuum. Then $pa^3 =$ the solidity of the globe, and $px^3 =$ the solidity of the vacuum; hence $pa^3 - px^3 =$ the solidity of the shell, and $bpa^3 - bpx^3 =$ its weight, as also the weight of the air that would be displaced by the globe is equal to cpa^3, therefore $bpa^3 - bpx^3 = cpa^3$, and $x = a \,.\, \sqrt[3]{(b - c)} \div \sqrt[3]{b} = 7998.3554$ miles, consequently $\frac{1}{2}(a - x) = .8223$ of a mile. Answer.

(7) *By* Master *W. Hebblethwaite.*

Required the distance between St. Helena, in lat. 15° 55′ S. lon. 5° 49′ W. and Halifax, in lat. 53° 47′ N. lon. 1° 52′ W.

By Master Charles Macauley,

In this question we have given the proper difference of latitude 4182, the meridional difference 4811, and the difference of longitude 237, whence the distance is found by Mercator's Sailing to be 4187 miles.

By Master Jonathan Homer.

In an oblique angled triangle ABC there are given, the complement of the latitude at Halifax 36° 13′ = AC, the latitude at St. Helena added to 90° = 105° 55′ = AB, and the difference of longitude = 3° 57 = $\angle$ A, to find BC; which by the 8th case of oblique spherical triangles (see Emerson's Trigonometry,) is found to be 69° 47′ = 4187 geographical, or 4849 .4106 English miles.

(8) *By* Philalethes, *Woodhouse-grove.*

To find two numbers such that their difference shall be a square, and if either of them be subtracted from the square of the other, the remainder shall be a square.

By Masters C. BARTHOLOMEW, and C. H. PIGOT.

Suppose the numbers to be x and y, then per question $x - y$, $x^2 - y$, and $y^2 - x$ must be squares. Assume $x = 2z + 1$, and $y = z^2 + 1$; then the first and third formulæ will be squares; but it remains to make $4z^4 + 3z^2$ or $4z^2 + 3 = \square$ (sup.) $= r^2$ and $z^2 = (r^2 - 3) \div 4$, or $r^2 - 3 = \square$ (sup.) $= (r - n)^2 = r^2 - 2rn + n^2$, and $2rn = 3 + n^2$, or $r = (3 + n^2) \div 2n$. Take $n = 1$, then $r = 2$ and $z^2 = \frac{1}{4}$. Therefore the numbers are $\frac{3}{2}$ and $\frac{5}{4}$.

By Master JAMES BRAMWELL.

Let x and y be the numbers; then by the question, $x - y$, $x^2 - y$ and $y^2 - x$ are all to be squares. Assume $x - y = \frac{1}{4}$; then $x = y + \frac{1}{4}$, and $x^2 - y = y^2 + \frac{1}{2}y + \frac{1}{16} - y = y^2 - \frac{1}{2}y + \frac{1}{16}$, which is evidently a square whatever y be, and $y^2 - x = y^2 - y - \frac{1}{4} = \square$, [put $= (n - y)^2$] $= n^2 - 2ny + y^2$; hence $y = (n^2 + \frac{1}{4}) \div (2n - 1)$; take $n = 1$, then $y = \frac{5}{4}$, and $x = \frac{6}{4}$, numbers that will answer.

Ingenious solutions to all the questions in the Youth's Department were transmitted by Masters W. Charnley, Pleasington School; S. Dowse, pupil at Mr. Hefford's Academy, Thornhill, near Wakefield; C. Macauley, Hope School, Liverpool; W. Hebblethwaite, pupil in Mr. Lewthwaite's Academy, Halifax; G. Aspden, Pleasington School; Jonathan Horner, Knaresbro'; W. Wilson, Knaresbro'; W. Armitage, Mr. Darby's Classical, Commercial and Mathematical Academy, Leeds; H. Ward and C. H. Pigot, pupils in Mr. Sheridan's Mathematical Academy, Stafford; J. Fairclough, Mr. Darby's Academy; R. Mounsey, Mr.

Huntington's academy, Preston; and C. Bartholomew, Woodhouse Grove seminary.

Master T. Butterworth, near Blackburn, answered 1, 2, 4.

Masters T. Fawcett and T. Lloyd, Mr. Darby's academy; T. Bamber, Mr. Huntington's academy; W. Pickup, Mr. Lightbown's seminary, Blackburn; W. B. Fuller, Eccleshill, and Michael Schofield, answered 1, 2, 3, 4, 5.

Hugh Huntington answered the 6th.

James Bramwell, Trinity Church School, Preston, answered 1, 2, 8.

W. Harewood, near Blackburn, answered 1,2,3,4,5,8.

B. Slack and W. Brownell, Woodhouse Grove Seminary, answered the 1st question.

ANSWER

TO THE

MATHEMATICAL QUESTIONS

Proposed in the Third Number of the Third Volume.

Six numbers of the Leeds Correspondent are now and will be hereafter regularly offered as a prize, to be adjudged BY LOT, to one Gentleman out of those who may give true solutions to the Prize Question in each number.

Mr. J. Wright, of Hull, who has answered the Prize Question in this Number, and on whom the lot fell for the Prize, will have the goodness to apply, either personally or by letter, to Mr. John Heaton, Bookseller, Briggate, Leeds, and he will receive six Copies of this Number gratuitously, as allotted to him.

(215) *By Mr. J.* Gillot, *Sheffield.*

Determine that arc, whose sine, tangent and secant are in harmonical ratio.

By Mr. John Baines, *East-field, Doncaster.*

Put x = the co-sine of the required arc, then $\surd(1 - x^2)$ = the sine, and $\frac{\surd(1 - x^2)}{x}$ and $\frac{1}{x}$ = the tangent and secant. By the nature of harmonical proportion, $\frac{1}{\surd(1-x^2)} + \frac{1}{x} = \frac{2x}{\surd(1-x^2)}$, or $2x - 1 = x\,\surd(1 - x^2)$ (i.e. twice the co-sine — the radius = the rectangle contained under the sine and co-sine); this reduces to $x^4 + 3x^2 - 4x + 1 = 0$; hence x = .748202, or — .340092, the natural co-sines of 41° 33′ 54″ and 109° 52′ 56″, which answer the question.

In solving the above equation by the rule given in the note commencing at p. 121 of Bonnycastle's Introduction to Algebra, in all the editions subsequent to the 10th, we have $a = 0$, $b = 3$, $c = -4$, and $d = 1$, from which we obtain the cubic equation $A^3 - \frac{3}{2}A^2 - A - \frac{1}{2} = 0$, and $A = \frac{1}{2} + \sqrt[3]{\left(\frac{5}{8} + \frac{1}{36}\surd 249\right)} + \sqrt[3]{(\frac{5}{8} - \frac{1}{36}\surd 249)} = 2.092193$. Now, Mr. B. says, (p. 123) "The value of A in the above equation is always commensurate and rational; not only when all the roots of the given equation are commensurate, but also when they are irrational and even impossible." This is not true; for the value of A in the above equation is irrational; and as the equation does not belong to the irreducible case of Cardan, we know that the other two roots are imaginary. Proceeding in the solution, we have B = 1.088295, and C = 1.837737; therefore $x^2 + 2.092193 = \pm 1.088295x + 1.837737$, or $x = \pm .544147 \pm \surd.0416385$ = .748202, or — .340092, as above.

By Mr. C. Waterhouse.

Let the sine be represented by S, and the secant by s, then the tangent will be $\frac{2Ss}{S+s}$. Now by a few trials we find the arc is about 41° 30′. (1) Therefore suppose the arc 41° 30′, the sine of which is .6626200 (= S), tangent .8847253, and secant 1.3351924 (= s), where $\frac{2Ss}{S+s} = .8856939$, which ought to be .8847253 making an error of + .0009686. Again, (2) Suppose 41° 34′ 5″ to be the arc, the sine, tangent and secant of which are .6635091 (S), .8868450, and 1.3365979 (s) respectively, in which place $\frac{2Ss}{S+s} = .8867974$, which ought to be .8868450, consequently the error is — .0000476.

1st Error + .0009686 1st Sup. 41° 30′
2d Error — .0000476 2d Sup. 41° 34′ 5″
Difference + .0010462 : 245″ :: .0000476 : 11″, and 41°34′5″−11″ = 41°33′54″, the answer nearly.

By Mr. T. Wilkinson, *Hull.*

Put x = sine of the required arc; then will $\frac{x}{\sqrt{[1-x^2]}}$ and $\frac{1}{\sqrt{[1-x^2]}}$ be tangent and secant, the radius being 1; and by the nature of harmonical ratio, we shall have $x : \frac{1}{\sqrt{[1-x^2]}} :: \frac{x}{\sqrt{[1-x^2]}} - x : \frac{1}{\sqrt{[1-x^2]}} - \frac{x}{\sqrt{[1-x^2]}} \therefore \frac{2x-x^2}{\sqrt{[1-x^2]}} = \frac{x}{1-x^2}$; whence by reduction $x^4 - 4x^3 + 3x^2 + 4x$

= 3, from which equation $x = .663467726$, the nat. sine of 41° 33′ 53″, the answer required.

Solutions were received from Messrs. Stringer, Lightbown, Huntington, Settle, Godward, Shepherd, Thomas, Wilkinson, Harker, Ryley and Clay.

(216) *By Mr. W.* GODWARD, *Sheffield.*

Given the perpendicular, the diameter of the circumscribing circle, and the ratio of the rectangle of the sides to the rectangle of the perpendiculars let fall from the angles at the base on the line bisecting the vertical angle, to construct the plane triangle.

By MR. H. LIGHTBOWN.

Let ABC be the required triangle, Am, Bn perpendiculars on the line CF bisecting the vertical ∠, HF the diameter of the circumscribing circle perpendicular to the base AB. Now AC × BC : Am × Bn :: $r : s$, a given ratio; but AC × BC = HF × DE, and Am × Bn = AO × BO, O being the point of contact of the inscribed circle, (See Gen. Comp. 1819) = EF × DE (Prop. 10, M. G.); that is HF × DE : EF × DE :: HF × EF :: $r : s$; but HF and DE are given, therefore EF is given, and consequently AB is given. Hence in a given circle it is required to inscribe a triangle whose base and perpedicular shall be given, a very simple problem.

By MR. WM. GODWARD, *Brampton Bull Head, near Barnsley.*

ANALYSIS. Imagine ABC to be the required △, whose circumscribing circle is ADCBE, having its diameter DE given, which bisects AB in K; CE the line bisecting the vertical angle; BH and AG perpendiculars drawn from the angles at the base upon the line CE; CF the perpendicular, and L the point of contact of the inscribed circle. Now $AC . BC = DE . CF$, (Simpson's Euc. B. VI. P.C) and $AG . BH = AL . LB$ (question 427, Mathematical Companion,) $= CF . KE$ (10 M. G. Student.) Moreover the ratio of $AG . HB$ to $AC . BC$ is given; therefore the ratio of their equals, i.e. $CF . KE$ to $DE . CF$ is given, and consequently that of KE to DE is given: But DE is given; hence KE is known. Therefore the construction is manifest.

Neat solutions to this question were given by Miss Harriet Bonny-Eye, and Messrs. Stringer, Windward, Huntington, Baines, Settle, Shepherd, Thomas, Wilkinson, Ryley and Clay.

(217) *By Mr. W. Stringer.*

Given the hypothenuse of a right angled triangle, to construct it when the sum of one leg and its alternate segment of the hypothenuse made by a perpendicular from the right angle is a maximum.

By the Proposer.

Analysis. Suppose the thing done, ACB the △ required, and CD perpendicular to AB in D. Then by the question $AC + DB$ is a maximum, or $AC + DB - AB = AC - AD$ a maximum; and by similar triangles $AC : AD :: AB : AC$, or $AC - AD : AC :: AB - AC : AB$; therefore $(AC - AD) . AB = (AB - AC) . AC$: but $(AC - AD) . AB$ is a maximum; $\because (AB - AC) . AC$ is so, which it is well known will be the case when $AC = \frac{1}{2}AB$. Whence the construction is obvious.

By Mr. W. Wright, *Stationer, Hull.*

Analysis. Imagine ACB the △ sought, right angled at C; draw CD perpendicular to AB; bisect

AB at O; and instead of a maximum, suppose $AD + BC$, or its equal $BC + AB - BD$, given; then AB being given, $BC - BD$ is given; in AB lay off $BE = BC - BD$, then $DE = BC$, and $DE^2 = DB^2 + BE^2 + 2BE \times BD = BC^2 = AB \times BD$; whence $AB \times BD - BD^2 - 2BE \times BD = BE^2$, or $(AB - BD - 2BE) \times BD = (AD - 2BE) \times BD = DF \times DB$ by making $AF = 2BE$; hence $DF \times DB = BE^2$. But B, F are given points, from which the following

CONSTRUCTION:—Upon the given hypothenuse AB take $AF = 2(BC - BD)$; upon BF as a diameter describe a semi-circle; at F erect FH perpendicular to AB, and $= \frac{1}{2}AF$; draw GHH′ parallel to AB, cutting the semi-circle at H, H′, through either of which draw the perpendicular CD; join A, B with C, and it is done. Consequently the problem admits of two solutions; and when GHH′, instead of cutting, only touches the semi-circle the maximum takes place, and it is pretty obvious that then $DB = DF = \frac{1}{2}AF = \frac{1}{2}OB = \frac{1}{4}AB$.

Ingenious solutions were transmitted by Miss Harriet Bonny-Eye, and Messrs. Lightbown, Huntington, Baines, Waterhouse, Settle, Godward, Shepherd, Thomas, Wilkinson, Harker, Ryley and Clay.

(218) *By Mr. H.* LIGHTBOWN, *Schoolmaster, Blackburn.*

To find three numbers in geometrical proportion such, that if any given number (n) be added to each of them, the three sums thence arising may all be rational squares.

By MR. J. BAINES.

Let $\frac{x}{s}$, x, and sx represent the three numbers in geometrical progression; then $\frac{x}{s} + n$, $x + n$, and sx

$+ n$ must be squares. Put $\frac{x}{s} + n = a^2$, and $x + n = b^2$, then $x = s(a^2 - n) = b^2 - n$, and $s = \frac{b^2 - n}{a^2 - n}$; hence $sx + n = \frac{(b^2 - n)^2}{a^2 - n} + n$ remains to be made a square. Put $a^2 - n = (b - a)^2 = b^2 - 2ab + a^2$, and $a = \frac{b^2 + n}{2b}$; hence $\frac{(b^2 - n)^2}{a^2 - n} + n = 4b^2 + n$; put it $= (ns - 2b)^2 = n^2s^2 - 4bns + 4b^2$, and $b = \frac{ns^2 - 1}{4s}$ in which s may be taken at pleasure.

Ex. If $s = 5$, and $n = 1$, we have $\frac{x}{s} = \frac{121}{3600}$, $x = \frac{11}{25}$, and $sx = \frac{144}{25}$, three numbers that will answer.

N.B. Another solution to this question, by the late Mr. Gawthorp of Leeds, may be seen in the Quarterly Visitor, Vol. II. p. 141.

By Mr. Wm. Shepherd, *Bradford.*

Let $4x^2$, $x^2 - n$, and $\frac{(x^2 - n)^2}{4x^2}$ be the numbers required; then $x^2 - n + n = x^2$, and $\frac{(x^2 - n)^2}{4x^2} + n = \frac{(x^2 + n)^2}{4x^2}$; hence it only remains to make $4x^2 + n$ a square $= (2x + p)^2 = 4x^2 + 4xp + p^2$; from which $x = \frac{n - p^2}{4p}$; where n and p may be any numbers, positive or negative. If $n = 21$, and $p = 1$, then $x = 5$; and the numbers are 100, 4, and $\frac{4}{25}$.

By Mr. W. Settle.

Let $\frac{x}{y}$, x and yx denote the numbers required; then per question $\frac{x}{y} + n$, $x + n$, and $yx + n$ must be made squares. Assume $x + n = a^2$, and $yx + n = b^2$, then $x = a^2 - n$, and $y = \frac{b^2 - n}{a^2 - n}$; and these values of x and y substituted in $\frac{x}{y} + n$ it becomes $\frac{(a^2 - n)^2 + (b^2 - n)^n}{b^2 - n}$, which must be a square; put $b^2 - n = 4a^2$, then will $\frac{(a^2 - n)^2 + (b^2 - n)^n}{b^2 - n} = \left[\frac{a^2 + n}{2a}\right]^2$ evidently be a square, and it only remains to make $b^2 - n = 4a^2$ a square; assume its root $b - r$, whence $b = \frac{r^2 + n}{2r}$, $a = \frac{r^2 - n}{4r}$, $y = \frac{4 . (r^2 - n)^2}{(r^2 - n)^2 - 16r^2n}$, $\frac{x}{y} = \frac{[(r^2 - n)^2 - 16r^2n]^2}{(8r)^2 . (r^2 - n)^2}$, $x = \frac{(r^2 - n)^2 - 16r^2n}{(4r)^2}$, and $yx = \frac{(r^2 - n)^2}{4r^2}$, where r and n may be taken at pleasure.

Ex. To find three numbers in geometrical progression, such that 1 being added to each, the three sums may be rational squares. In this example $n = 1$, and take $r = 5$, then $\frac{x}{y} = \frac{[(r^2 - n)^2 - 16r^2n]^2}{(8r)^2 . (r^2 - n)^2} =$

$\frac{121}{3600}$, $x = \frac{(r^2 - n)^2 - 16r^2n}{(4r)^2} = \frac{11}{25}$, and $yx = \frac{(r^2 - n)^2}{4r^2} = \frac{144}{25}$, which three numbers will be found, on trial, to answer the conditions of the question.

By changing the signs in the above expressions for $\frac{x}{y}$, x and yx, wherein n is concerned, we shall obtain a solution to the following question:—To find three numbers in geometrical progression such, that if a given number n be subtracted from each of them, the three numbers thence arising may be squares.—As in the above example let $n = 1$, and $r = 4$, then $\frac{x}{y} = \frac{[(r^2 + n)^2 + 16r^2n]^2}{(8r)^2 . (r^2 + n)^2} = \frac{(545)^2}{1024 \times 289}$, $x = \frac{(r^2 + n)^2 + 16r^2n}{(4r)^2} = \frac{545}{256}$, and $xy = \frac{(r^2 + n)^2}{4r^2} = \frac{289}{64}$, which three numbers will be found on trial, in this case, to answer the conditions of the question.

Ingenious solutions to this question were given by Miss Harriet Bonny-Eye, and Messrs. Stringer, Windward, Lightbown, Huntington, Waterhouse, Godward, Wilkinson, Harker and Clay.

(219) *By Mr.* ANTHONY COOK, *Wooley near Hexham.*

AB and AC are given chords of a given circle ACB; it is required to draw BED cutting AC in E and the circle in D, so that joining AD the area of the triangle ADB may be equal to the rectangle ED. DB.

By MR. JESSE WINDWARD *and* MR. S. RYLEY.

GEOM. ANALYSIS. Admit the problem solved, and the line BED drawn as required; draw AF perpendicular to BD, then, per question and obvious principles, (Q.E.D.) DB = DB . AF, or AF : ED :: 2 : 1, a given ratio; and the ∠ EDA is also given, because BA is a given chord in a given circle, ergo the △ AED is given in species; therefore the ∠ AED, and consequently its supplement AEB, is given, and the locus of E a given circle; therefore the method of drawing BD is obvious.

Messrs. Stringer, Lightbown, Huntington, Waterhouse, Settle, Godward, Shepherd, Thomas, Wilkinson, and Miss Harriet Bonny-eye gave true solutions to this question.

(220) *By Mr. John Baynes Jun.*

$(\sqrt{-1})^{\sqrt{-1}}$ is a constant quantity. Required a proof.

By the Proposer.

I. Log. $\left\{(\sqrt{-1})^{\sqrt{-1}}\right\} = \sqrt{-1} \times \log. \sqrt{-1}$.

II. Put $\frac{1+x}{1-x} = \sqrt{-1}$, and we get $x = \sqrt{-1}$.

III. Hyp. log. $\frac{1+x}{1-x} = 2 \times (x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \frac{1}{7}x^7 + \&c.) =$ by substitution $2 \times \{(\sqrt{-1}) + \frac{1}{3}(\sqrt{-1})^3 + \frac{1}{5}(\sqrt{-1})^5 + \frac{1}{7}(\sqrt{-1})^7 + \&c.\}$

IV. From I, $2 \times \sqrt{-1} \times \{(\sqrt{-1}) + \frac{1}{3}(\sqrt{-1})^3 + \frac{1}{5}(\sqrt{-1})^5 + \frac{1}{7}(\sqrt{-1})^7 + \&c.\}$

V. By actually multiplying IV, $2 \times \{(\sqrt{-1})^2 + \frac{1}{3}(\sqrt{-1})^4 + \frac{1}{5}(\sqrt{-1})^6 + \frac{1}{7}(\sqrt{-1})^8 + \&c.\}$

VI. By squaring each term in V, $2 \times \{(-1) + \frac{1}{3}(-1^2) + \frac{1}{5}(-1^3) + \frac{1}{7}(-1^4) + \&c.\}$

VII. By involution VI, $2 \times (-1 + \frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \&c.)$

VIII. Changing the signs, $-2\,(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \&c.)$ Now because the series within the parenthesis is $=$ .78539816, we have

IX. $\sqrt{-1} \times \log.\ \sqrt{-1} = -2 \times .78539816 = -1.57079632 =$ the hyp. log. of .2078796, the Answer.

N.B. The least positive quantity, as x^x is when $x = h.l.{}^{-1}$; all other values of x will give a greater. This solution involves the quadrature of the circle in fact, if the diameter of a circle be 1, its circumference is $\frac{4}{\sqrt{-1}} \times h.l.\ \sqrt{-1}$.

(221) *By Mr. H. Lightbown.*

In abbreviating or reducing fractions to their lowest terms, it is usual to find a common measure, by dividing the greater term by the less, the divisor by the remainder, till nothing remains. Prove universally, that the last divisor of all is the greatest common measure.

By the Proposer.

Let $\frac{N}{D}$ denote any proper fraction whatever, or one whose numerator is less than its denominator, and in dividing as expressed in the question, let the respective quotients be Q, Q′, Q″, Q‴, &c. and the remainders R, R′, R″, &c. that is

$$(1) \qquad \begin{array}{r|l} N\,)\ D & (\,Q \\ \underline{N \times Q} & \\ 0 & \end{array}$$

Then since $D = N \times Q$, therefore $\frac{N}{D} = \frac{N}{N \times Q} = \frac{1}{Q}$; and consequently in this case N is the greatest amon measure of the fraction $\frac{N}{D}$, and $\frac{1}{Q}$ is the ction in its lowest terms.

2)

$$\begin{array}{l} N\,)\,D\quad(Q \\ \quad\underline{N\times Q} \\ \quad R\,)\,N\quad(Q' \\ \qquad\underline{R\times Q'} \\ \qquad\quad 0 \end{array}$$

Then $N = R\times Q'$, and $D = N\times Q + R$; hence $= \frac{R\times Q'}{N\times Q+R} = \frac{R\times Q'}{R\times Q\times Q'+R} = \frac{Q'}{Q\times Q'+1}$ R is the greatest common measure of the fraction , and $\frac{Q'}{Q\times Q'+1}$ is the fraction in its lowest rms.

(3)

$$\begin{array}{l} N\,)\,D\quad(Q \\ \quad\underline{N\times Q} \\ \quad R\,)\,N\quad(Q' \\ \qquad\underline{R\times Q'} \\ \qquad R'\,)\,R\quad(Q'' \\ \qquad\quad\underline{R'\times Q''} \\ \qquad\qquad 0 \end{array}$$

Then we have $R = R'\times Q''$, $N = R\times Q'+R'$, and D : $N\times Q+R$; therefore $\frac{N}{D} = \frac{R\times Q'+R'}{N\times Q+R} =$
$\frac{R\times Q'+R}{(R\times Q'+R')+R} = \frac{R'\times Q'\times Q''+R'}{R'\times Q\times Q'\times Q''+R'\times Q+R'\times Q''}$
: $\frac{Q\times Q''+1}{Q\times Q'\times Q''+Q+Q''} = \frac{Q'\times Q''+1}{Q\times(Q'\times Q''+1)+Q''}$; nsequently R' is the greatest common measure of e fraction $\frac{N}{D}$, and $\frac{Q'\times Q''+1}{Q\times(Q'\times Q''+1)+Q''}$ is the action in its lowest terms.

In the next case if we suppoſe the number of quotients to be four, viz. Q, Q', Q'', Q''', and the last remainder R'', we should get $\frac{N}{D} =$

$$\frac{Q' \times (R'' \times Q'' \times Q''' + R'') + R'' \times Q'''}{Q \times (Q' \times Q'' \times Q''' + R'' \times Q' + R'' \times Q''') + R'' \times Q'' \times Q''' + R''}$$
$$= \frac{Q' \times Q'' \times Q'' + Q' + Q'''}{Q \times (Q' \times Q'' \times Q''' + Q' + Q''') + Q'' \times Q''' + 1},$$

or the last remainder R'' is the greatest common measure of the fraction $\frac{N}{D}$, and therefore

$$\frac{Q' \times Q'' \times Q''' + Q' + Q'''}{Q \times (Q' \times Q'' \times Q''' + Q' + Q''') + Q'' \times Q''' + 1}$$

is the fraction in its lowest terms, &c. &c. universally. Q.E.D.

By MR. J. BAINES *and* MR. S. RYLEY.

Let $\frac{b}{a}$ be the given fraction, and let $a + b = c + \frac{d}{b}$, $b \div d = e + \frac{f}{d}$, $d \div f = g$, &c.; but in the present case, suppose the work to terminate at g, and that f is the last divisor. Now, we have the following equations, viz. $a = bc + d$, $b = cd + f$, and $d = fg$; hence, by substitution $a = bc + fg = c^2d + cf + fg = c^2fg + cf + fg$, and $b = cfg + f$; therefore $\frac{a}{b} = \frac{c^2fg + cf + fg}{cfg + f} = \frac{c^2g + c + g}{cg + 1}$, and hence the greatest common measure is f.

Ingenious answers to this question were given by Messrs. Stringer, Huntington, Waterhouse, Settle, Godward, Shepherd, Thomas, Ryley and Clay.

222) *By Mr. J.* Harker, *Leeds.*

ts lying east and west of each other are 200 ıder. A ship sails from the most eastern he west and north: Another sails from the ern between the east and north, both taking ırses till they meet, and after comparing their sailed, found that the one from the most ort had sailed 25 miles more than the other; the nearest distance between the place where et, and a direct line between the two ports, to niles, required each ship's course, distance run, ference of latitude.

y Harriet Bonnyeye, *Woodhouse Grove.*

B and E represent the two ports sailed from the place where the ships meet. Then in the gle BEC (here considered a plane one,) we have n the base BE = 200, the perpendicular AC = and BT = 25 the difference of the sides BC, CE etermine them. This problem has often been lved; but the following analysis which suggests a y simple construction is entirely new:

From the centres B and C, with the radii BT and T = CE, describe the circles BT*m* and ETFG, and t PEF be drawn perpendicular to BE meeting PT, common tangent to them, in P; produce BC to meet he circle, whose centre is C, in G; and join PB, ET, G. Then EF $= 2AC = 172$, and by the circle $PE . PF$ or $PE^2 + PE . EF = PT^2 = PB^2 - BT^2 = BE^2 + PE^2 - BT^2$. Whence $PE = (BE^2 - BT^2) \div EF = (200^2 - 25^2) \div 172 = 228.9244$, and therefore $PT = \sqrt{PE . PF} = 302.9544$.

Again, since the points P, E, T, B are in a circle in consequence of PTB, PEB being right angles, we have BE . PT = PB . ET + PE . BT; or obviously ET = (BE . PT — PE . BT) ÷ PB. Now the triangles GET and BEP are similar; for the angles GTE and BPE are equal, and GET, BEP are right angles; hence PE : PB :: ET : GT = $\frac{\text{BE . PT}}{\text{PE}}$ — BT = $\frac{200 \times 302.9544}{228.9244}$ — 25 = 239.676, the half of which is CT or CE = 119.838; this + 25 gives BC = 144.838, the distances run by the two ships; from which the courses are readily found to be 36° 25′ and 45° 52′ nearly.

By MR. W. GODWARD, *and* MR. C. THOMAS, *Eccleshill.*

Let E and W represent the Eastern and Western ports respectively, N the port sailed to, NA perpendicular to EW will be the shortest distance. Put x = NW, then $x + 25$ = EN; by the question and 47. E.1, we have $(x^2 + 50x - 6771)^{\frac{1}{2}} + (x^2 - 7396)^{\frac{1}{2}} = 200$, which reduces to this quadratic $x^2 + 25x = \frac{4374001}{252}$, solved gives $x = 119.838$ miles, the distance sailed by the Western ship; then $x + 25 = 144.838$ miles, the distance sailed by the Eastern ship.

Again, by Trigonometry, EN : NA = 86 :: 1 = rad : .593766, the nat. sine ∠ AEN = 36° 25′ 29″ the course of the Eastern ship; also, by Trigonometry, WN : NA :: 1 (rad.) : .717635, the nat. sine ∠ AWN = 45° 51′ 34″, the course of the Western ship.

Solutions were received from Messrs. Stringer, Baines, Huntington, Settle, Shepherd, Thomas, Wilkinson, Ryley, Eley and Clay.

(223) *By Mr. C. Holt, Chorley.*

l by the centrobaric method the content of the
ormed by the revolution of a quadrant of a cir-
ose radius is 20 about its tangent.

By Mr. W. Godward.

the outlines of the diagram represent a section
generated solid; put radius 20 $= a$, .7854 $= p$,
at. sine of the $\angle$ ADF ($= 45°$) $= s$; then by
ium to prop. 47, Emerson's Mechanics, arch
$p) :$ chord $(a\sqrt{2}) :: \frac{2a}{3} : \frac{a\sqrt{2}}{3p} =$ AD, the point
ing the centre of gravity of the quadrant.
;ain, by Trig. radius $= 1 : s :: \frac{a\sqrt{2}}{3p} : \frac{as\sqrt{2}}{3p} =$
then DE $=$ AC $-$ 2AF $= 2a - \frac{2as\sqrt{2}}{3p} = \frac{2a}{3p}$
$- s\sqrt{2})$, therefore $\frac{2a}{3p}(3p - s\sqrt{2}) \times 4p = \frac{8a}{3}$
$- s\sqrt{2}) =$ the circumference described by the
t D; also $a^2p =$ area of the generating quadrant.
Barlow's Dictionary, art. Centro-baryc, we have $\frac{8a}{3}$
$- s\sqrt{2}) \times a^2p = \frac{8a^3p}{3}(3p - s\sqrt{2}) = 22723$
24 the solid content required, which is equal to
solidity of a hemisphere whose radius is a, multi-
d by $3p - s\sqrt{2} = 1.3562$.

By Mr. John Huntington, *Preston.*

et AOBC denote any quadrant, AT the tangent
it which it revolves, and OT a line drawn through

the centre of gravity g, and meeting the tangent AT in T; Draw gr parallel to AO to meet AT in r; put the radius AO of the quadrant $= a$, and .78539 &c. $= n$. Then $\text{OT} = \text{AB} = \text{AO}\sqrt{2} = a\sqrt{2}$, the quadrantal arc $\text{AB} = 2an$, and the generating area AOBC $= a^2n$. Now it is well known that $\text{OG} = \dfrac{a\sqrt{2}}{3n}$; therefore $\text{T}g = \text{OT} - \text{OG} = \dfrac{3n - 1}{3n} \times a\sqrt{2}$, $rg = \dfrac{a}{3n}(3n - 1)$, and the circumference of a circle to the radius $rg = \dfrac{8a}{3n}(3n - 1)$. But the generating area multiplied by the circumference of the circle described by the centre of gravity = the solid content. Therefore $a^2n \times \dfrac{8a}{3}(3n - 1) = \dfrac{8a^3n}{3}(3n\text{-}1)$, the solidity in any given case.

Assume $a = 20$, and restore the value of n; then $\dfrac{8a^3n}{3}(3n - 1) = 22723.25678$, the required content of the solid.

By MR. W. SETTLE, *Leeds, and* MR. W. ELEY, *Rolleston Grammar School.*

The distance of the centre of gravity of the given quadrant from the revolving axe, is readily found to be 11.5118, and $11.5118 \times 6.283186 = 72.33078$, is the circumference described by that centre; also the area of the generating plane is $\dfrac{40 \times 40 \times .7854}{4} = 314.1592$. Therefore $314.1592 \times 72.33078 = 22723.37998$ is the content of the solid required.

Messrs. Eley and Clay also transmitted true solutions to this question.

AA

(224) *By Mr. W.* Settle, *St. James's Street, Leeds.*

A cabinet-maker having a triangular plank, whose three sides are 27, 36 and 45 inches, wishes to cut two elliptical tea-boards out of it, in the ratio of two to one, and the largest possible. Please to inform him what will be their dimensions.

By Plus Minus.

Lemma. The greatest ellipse that can be inscribed in a given triangle, bisects the sides in the points of contact, has its centre in the centre of gravity of the triangle, its centre and circumference trisect each of the straight lines drawn from the angles to the points of contact, and its area is to the area of the triangle as 3.1416 to $3\sqrt{3}$.

This Lemma is proved analytically by M. Berard, principal and professor of Mathematics in the College of Briançon, in vol. IV. of *Annales de Mathematiques.* But perhaps the most simple method of proving it is this. An equilateral triangle is the least triangle that can be described about a given circle. Suppose that an equilateral triangle and its inscribed circle are projected on a plane by rays issuing from a given point, the projection will be a triangle with its greatest inscribed ellipse; and because the sides of the equilateral triangle are bisected by the points of contact of the inscribed circle, the sides of the triangular projection will evidently be divided in the same ratio. Also since lines drawn from the angles of the equilateral triangle to the points of contact, intersect each other in the centre of the circle, or the centre of gravity of the triangle, and are each trisected by the centre and circumference of the circle, it follows, that the lines drawn from the angles of the projected triangle to the points of contact of the inscribed ellipse will all intersect in the centre of the ellipse, or centre of gravity of the triangle, and be trisected by the centre and circumference. And lastly, the area of the ellipse will be to the area of the triangle, as the area of the circle to the area of the circumscribing equilateral triangle, which is easily shewn to be as 3.1416 to $3\sqrt{3}$.

This being premised, the solution of the problem may be as follows:—

Let XYZ represent the triangular plank, ARBGA and GIKVG the two ellipses to be cut out of it touching the sides in A, R, K, V, and the line YU in G; then by the Lemma, RX = RY, AX = AU, GY = GU, KY = KZ, and VZ = VU, when the ellipses are the greatest possible: also tri. XYU : tri. UYZ :: ellipse ARGA : ellipse GKVG :: XU : UZ :: (per question) 2 : 1; whence UZ = $\frac{1}{3}$ XZ. Draw YA bisecting XU in A, and take AE = EB = $\frac{1}{3}$AY, then E will be the centre of the ellipse and the diameter AB = $\frac{2}{3}$AY; from B draw BS parallel to AX, cutting XY in S; then by Conics, the square of CE, the semi-conjugate, to AB is equal to the rectangle AX . BS; but BS = $\frac{1}{3}$AX, therefore $CE^2 = \frac{1}{3}AX^2$.

Hence if a and b denote the semi-axes major and minor, we have by Conics $a^2 + b^2 = \frac{1}{9}AY^2 + \frac{1}{3}AX^2$, and by the Lemma ab = area of the triangle XYU ÷ $3\sqrt{3}$. Draw Yn perpendicular to XZ, then $Yn = XY . YZ \div XZ$ (because the angle XYZ is right,) $= 36 \times 27 \div 45 = 21.6$, $Zn = YZ^2 \div XZ = 27^2 \div 45 = 16.2$. Therefore $An = 13.8$, AX = AU = UZ = $\frac{1}{3}$XZ = 15: then Eu. 47.1, $AY^2 = Yn^2 + An^2 = 657$; $\therefore a^2 + b^2 = \frac{1}{3}AX^2 + \frac{1}{9}AY^2 = 148$, $\triangle XYU \div 3\sqrt{3} = 36\sqrt{3} = ab$.

Hence the transverse diam. or $2a = 2\sqrt{[37+18\sqrt{3}]} + 2[\sqrt{37 - 18\sqrt{3}}] = 21.340076$, and conj. $2b = 2\sqrt{[37 + 18\sqrt{3}]} - 2\sqrt{[37 - 18\sqrt{3}]} = 11.687648$.

Lastly VZ = $\frac{1}{2}$UZ = 7.5. $\therefore nV = 8.7$, $VY^2 = 542.25$, and $\triangle YUZ \div 3\sqrt{3} = 18\sqrt{3}$; hence we obtain the *principal* diameters of the ellipse GIKV, viz.

Transverse diam. $= \sqrt{[79 + 36\sqrt{3}]} + \sqrt{[79 - 36\sqrt{3}]} = 15.96921$. Conjugate diam. $= \sqrt{[79 + 36\sqrt{3}]} - \sqrt{[79 - 36\sqrt{3}]} = 7.80925$.

COR. The rectangle contained by the principal semi-axes of the greatest ellipse that can be inscribed in a given triangle, is equal to the area of the triangle divided by $3\sqrt{3}$; and the sum of their squares is equal one ninth of the square of the line bisecting the base, together with one third of the square of the semi-base.

A true solution was received from Mr. Huntington.

(225) *By Mr. J. North.*

Prove that if two opposite angles of a trapezium be equal, the sides cannot be in arithmetical progression.

By the Proposer and MR. WM. GODWARD.

Let a, b, c, d denote the sides of the trapezium, and e the angle included between the sides a and b, or c and d; then by Trigonometry the square of the diagonal opposite the angle e will be $a^2 + b^2 - 2ab.$ cosine $e = c^2 + d^2 - 2cd.$ cosine e. Therefore cosine e

$$= \frac{c^2 + d^2 - a^2 - b^2}{2(cd - ab)}.$$

If instead of a, b, c, d, in this formula, we substitute the arithmeticals $a', a' + n, a' + 2n, a' + 3n$, there will result cosine $e = 1$, or sine $e = 0$; indicating that it is impossible for two opposite angles of a trapezium to be equal when its sides are in arithmetical progression.

By MR. JESSE WINDWARD, *West Houghton, near Bolton le Moors, Lancashire, and* MR. HENRY CLAY, *Gedney Dyke, Lincolnshire.*

DEMONSTRATION. Let ABCD be a trapezium, of which the opposite angles A and C are equal, we have to prove that the sides AB, BC, CD and DA cannot be in arithmetical progression, or that the equality AB + AD = BC + CD does not exist. Produce DC and DA to E and F, making CE = CB, and AF = AB; join BF, BE and EF, and let BC be greater than AB; then, obviously, DF = AB + AD, and DE = BC + CD; because the angles C and A are equal, the isosceles $\triangle$s CBE, ABF are similar. Therefore angle CEB = angle AFB; but BC is greater than BA, ergo BE is greater than BF, consequently the $\angle$ BFE is greater than the $\angle$ BEF; to the equal angles BFA, BEC add the angles BFE, BEF, and the angle EFD will be greater than the angle FED, and consequently DE greater than DF; that is, BC + CD greater than AB + AD. Q.E.D.

By MR. WM. WRIGHT, *and* MR. S. RYLEY.

DEMONSTRATION. Let ABCD be any trapezium, in which the $\angle$ D = $\angle$ B; through A, B, C describe a circle; bisect the arc AFC at F; draw BF parallel to CD, AD respectively; draw AE, CE meeting at E; then by pars. the $\angle$ AEC = $\angle$ ADC = $\angle$ ABC by the question; draw EF also FG perpendicular to AB, and FH to AE.

Now, in the case of an arithmetical progression, the sum of the means AB, BC is equal to the sum of the extremes AE, CE; but 2BG = AB + BC, and 2EH = AE + EC = CD + AD, therefore BG and EH ought to be equal, and thence by similarity of triangles FG, FH ought to be equal; but the points A, G, H, F are in the periphery of a semi-circle, therefore the chords AG, AH cannot be equal, consequently 2BG, 2BH are unequal, and thence their equals AB + BC and AE + EC = CD + AD cannot be equal; and therefore the four sides of the trapezium cannot be in arithmetical progression. Q.E.D.

We may also readily shew, that, if two opposite angles of any trapezium be equal, the four sides cannot be in *geometrical progression.*

Solutions were received from Messrs. Stringer, Huntington, Settle, Shepherd, Thomas, Wilkinson and Clay.

(226) *By* AMICUS.

Two objects of the same known height (2 yds.) stand at given horizontal distances (60 and 80 yds.) from the bottom of a tower, and appear to a person at the top of it also of the same height. Find the tower's height.

By MR. WM. SHEPHERD, *and* MR. CHARLES THOMAS.

Make the rectangle ABDC so that its length AC or BD shall be equal to the sum (140) of the given distances, and its breadth AB or CD equal to (2) the given height of the objects.

Draw the diagonal BC, bisect it in O, and about O as a centre with the radius OB or OC describe the circle ABPDC; take AE = (80) one of the given distances, and at the point E erect the perpendicular ordinate EP; then EP is the height of the tower.

For since BC is the diameter of the circle, and the angles BAC, BDC right angles, the points A, B, D, C are in its circumference; and because the chords AB, CD are each equal to the given height of the objects, the corresponding arcs, and consequently the subtended $\angle$ s APB, CPD are equal: therefore by the principles of optics, the objects AB, CD will appear to an eye at P of the same height, the horizontal distances AE, CE being equal to those given by construction.

COMPUTATION. Draw OF, OG perpendicular to AC, PE; then $OG = FE = \frac{1}{2}(AE - EC) = 10$, $GE = OF = \frac{1}{2}CD = 1$, $OP^2 = OC^2 = FC^2 + FO^2 = 70^2 + 1^2 = 4901$, and $GP = \sqrt{[OP^2 - OG^2]} = \sqrt{4801} = 69.2892$, to which add $EG = 1$, the sum gives $EP = 70.2892$ yards, the height of the tower.

By Mr. John Huntington.

Let AB represent the tower, CD and EF the two equal objects at any given distances AC and AE from the bottom of the said tower; join BC, BD, BE, BF and DF, and extend FD to meet AB in G; demit DH and FI perpendicular to BC and BE respectively; assume AC $= a$, AE $= b$, CD = EF = AG $= c$, and BG $= x$; then AB $= x + c$, and by the prob. of right angled triangles BC $= \sqrt{[(x+c)^2 + a^2]}$, BD $= \sqrt{x^2 + a^2}$, BE $= \sqrt{[(x+c)^2 + b^2]}$, and BF $= \sqrt{[x^2 + b^2]}$.

Therefore, by similar triangles, BC : AC :: CD : DH $= \dfrac{ac}{\sqrt{[(x+c)^2+a^2]}}$, and BE : AE :: EF : FI $= \dfrac{bc}{\sqrt{[(x+c)^2+b^2]}}$. But since the visive angles CBD and EBF are equal by the question, the right angled triangles BDH and BFI are similar; hence BD : DH :: BF : FI. That is, $\sqrt{[x^2+a^2]} : \dfrac{ac}{\sqrt{[(x+c)+a^2]}} :: \sqrt{[x^2+b^2]} : \dfrac{bc}{\sqrt{[(x+c)^2+b^2]}}$; $\therefore \dfrac{bc\sqrt{[x^2+a^2]}}{\sqrt{[(x+c)^2+b^2]}} = \dfrac{ac\sqrt{[x^2+b^2]}}{\sqrt{[(x+c)^2+a^2]}}$, or by reduction $x^2 + cx = ab$.

Consequently, $x = \pm\sqrt{[ab + \frac{1}{4}c^2]} - \frac{1}{2}c$, and $x + c = \pm\sqrt{[ab + \frac{1}{4}c^2]} + \frac{1}{2}c$, a general expression for the height of the tower.

Ex. By adopting the numbers given in the question, viz. $a = 60$, $b = 80$, and $c = 2$ yards, we have $\pm\sqrt{[ab + \frac{1}{4}c^2]} + \frac{1}{2}c = \sqrt{4801} + 1$ (the other value being negative) $= 70.2892488$ yards, the required height of the tower.

Neat Geometrical and Algebraical solutions to this question were transmitted by Messrs. Stringer, Baines, Settle and Ryley.

(227) *By* IOTA.

As the gravity of bodies must be diminished at the equator by two causes, by the equatorial surface being 17 miles further from the centre than the circumpolar, and by the increase of the centrifugal force; required how much lighter the cargo of an East India ship, burden 1000 tons, will be at the equator than when she left the port of London.

By PHILO-VERITAS.

As the figure of the earth is not given, we will suppose it to be an oblate spheroid of which the axis is to the equatorial diameter as 229 to 230, being the form ascribed to it by Sir Isaac Newton. Princip. Lib. III. Prop. 19.

In the spheroid thus constituted, the gravitation at the equator is to that at the pole as 230 : to 231, and generally the gravitation g at the equator is to the gravitation g' at any place of which the latitude is l as 230 : to $230 + \sin^2 l$. (Vide Art. 302, vol II. Playfair's Outlines of Natural Philosophy.)

Hence taking $l = 51° 32'$, or $\sin^2 l = .61304$, we have in the present case this proportion:

As 23061304 : 23000000 :: 1000 tons : 997.341694 tons = weight of the ship at the equator. Consequently she would be lighter by 2 tons, 13 cwt. 18lbs. 9oz. at the equator than when she was at London.

By MR. WM. SETTLE, *St James Street, Leeds, and* MR. T. WILKINSON, *Hull.*

Sir Isaac Newton asserts in his Principia, Book III. Prop. 20, that the increase of gravity in passing from the equator to the pole is nearly as the versed sine of double the latitude (or as the square of the right sine of the latitude,) and the excess of the force of gravity at the pole to the force of gravity at the equator as 1 to 229: And therefore since the latitude of London is 51° 32', that of the equator 0°, and that of the pole 90°, the versed sines of double those arcs will be

12430507 and 20000000, the radius being 10000000, and the excess of the force of gravity at the pole to the force of gravity at the equator as 1 to 229, the excess of the force of gravity at London will be to the force of gravity at the equator as $1 \times \frac{12430507}{20000000}$ to 229, or as 12430507 to 4580000000; and therefore 4580000000 : 12430507 :: 1000 : 2 tons 14 cwt. 1qr. 3lb. 8oz, the difference of weight required.

True solutions were also given by Messrs. Godward and Ryley.

(228) *By Mr. A.* ROBINRATE, *Dublin.*

If a tube be fastened to the arm of a gin, such as those employed in raising coals from the pit, and carried round with a velocity equal to that of hail falling during a perfect calm, could such a position be assigned to the tube as that a hailstone might pass through it without touching the sides? If so, what is that position?

By MR. JESSE WINDWARD, *and* MR. W. SETTLE.

From a little consideration of this problem, it is pretty obvious that the position of the tube cannot be a straight line, because one motion is circular and the other a rectilineal one; therefore, instead of the tube being fixt to the arm, let us conceive it fixt to the gin itself, which is a cylinder; and let ABCD represent a portion of the cylinder when stretched out upon the horizon, and AC the tube; then obviously AB represents the velocity of the gin and CB that of the hailstone, which by the question are equal and uniform.

Therefore AB = BC, and ∠ CAB = ½ a right ∠; ergo, if the tube be fixt to the side of the cylinder making ½ a right angle with the horizon, the required position is assigned. It is also pretty obvious that the tube will form a portion of the Archimedean Spiral.

(229) *By Mr. J. H.* Swale, *Brunswick Place Academy, Liverpool.*

To describe a triangle given in species, having its angles posited in the circumferences of three given concentric circles.

By the Proposer.

Let EFD be similar to the required triangle; and Q the centre of the given circles QC, QA, QB.

Draw any radius QC; make the angle QCP = DFE, and QC : CP = DF : FE; to the circle QA apply PA, so that FD : FE = QB : PA; draw CA, CB, AB, making the angle ACB = PCQ: the required triangle is ACB.

Draw QA, QB.

By the Comp., CQ : CP = FD : FE = QB : PA, or alternately, CP : PA = CQ : QB. Also the angle QCB = PCA, for PCQ = ACB.

Therefore, PCA, QCB are equiangular triangles, having PC : CA = QC : CB, and alternating, PC : CQ = AC : CB = EF : FD, by the Composition.

Whence, as the angle ACB = PCQ = EFD, the triangles ACB, EFD are mutually equiangular; or ACB is the triangle required.

Ingenious solutions to this question were also received from Messrs. Stringer, Wright, Windward, Huntingdon, Settle and Wilkinson.

(230) *Prize Question, By Mr. C.* Green, *Greenwich Observatory.*

Observed the sun's true altitude, at Greenwich Observatory, to be 45° 41′, and one hour after I found it increased by just 8° 4′. Required the day and hours when these observations were made?

N. B. This question is answered algebraically in another place; but it is here reproposed to be done geometrically.

By the Proposer and Mr. William Ringhall, *whose solutions are nearly identical.*

Construct the isosceles spheric triangle PZE so that the two sides PZ, PE shall be each equal to 38° 31′ 20″ the co-latitude of Greenwich, and the included angle ZPE equal to 15°, the degrees in the given interval of time between the observations; and from the centres E, Z, with the radii ES, ZS equal to the sun's co-altitudes 44° 19′ and 36° 15′ at the first and second observations respectively, describe arcs to intersect in the point S; then draw the great circle PS, which will be the sun's co-declination, and the angles SPE, SPZ the hour angles from noon at the times required.

For draw S′P = SP, making the angle S′PZ = SPE, and join S′Z. Then because in the triangles PZS′, PES the two sides PS′, PZ are respectively equal to the two sides PS, PE, and the included angles S′PZ and SPE are also equal, the triangles are identical or equal in all respects: therefore S′Z = SE, and consequently S′Z, SZ are equal to the given co-altitudes of the sun at the times of observation; and taking away from each of the equal angles S′PZ, SPE the angle SPZ common to both, there remains the angle S′PS = ZPE, the given angle by construction; also since PZ is the co-latitude, P will represent the pole, Z the zenith of Greenwich, and S′, S the places of the sun at the times of observation, supposing his declination to remain constant during the given interval. The computation is as follows:

Draw P*n* bisecting ZE at right angles in *n*.

Then in the right angled spheric triangle P*n*E are given PE = 38° 31′ 20″ and the angle *n*PE = 7° 30′, to find E*n* = 4° 39′ 46″ and the $\angle$ PE*n* = 84° 7′ 10″. Consequently ZE = 9° 19′ 32″. Now in the oblique spheric triangle ZES all the sides are given, viz. ZS = 36° 15′, ES = 44° 19′, and EZ = 9° 19′ 32″ to find the angle SEZ = 27° 31′ 43″; which added to the $\angle$ PE*n* = 84° 7′ 10″, gives the $\angle$ SEP = 111° 38′ 53″.